

# Bus Energy Minimization by Transition Pattern Coding (TPC) in Deep Sub-Micron Technologies

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## Abstract

The energy dissipation associated with driving long wires accounts for a significant fraction of the overall system energy. This is particularly the case with the increasing importance of the inter-wire parasitic capacitance in deep sub-micron technology. A closed form solution for estimating the energy dissipation of a data bus is presented that uses an elaborate parasitic wire model. This includes the distributed RLC effects of wires as well as the coupling between wires. We also propose a general class of coding techniques to reduce energy dissipation for data transmission by trading-off between computation and communication costs. An algorithm is presented to design efficient coding strategies to minimize energy. When the effects of inter-wire capacitance are taken into account, the best coding strategy is not to simply minimize transitions - an approach followed by previous research. Instead, Transition Pattern Coding (TPC) modifies the transition profile to minimize energy, and in many cases higher transition activity can result in lower energy. Results show that up to a factor of 2 reduction in energy is achieved through data coding.

## 1. Introduction

As technology scales to the deep sub-micron dimensions, the energy cost of performing computation continues to reduce while the cost of on-chip communication is not improving. Over the past several years, significant emphasis has been placed on reducing the energy dissipation associated with communication. Numerous schemes have been presented for reducing energy associated with driving wires including low swing signaling, charge re-cycling and data coding [5][7][8]. These techniques have assumed a simplistic lumped model for the wire.

One effective coding technique to reduce switching energy under the lumped model is the bus-invert technique in which the data bus is conditionally inverted to reduce the overall transitions. If more than 50% of the bits change, the entire bus is inverted. Therefore, in addition to the data, an extra bit must be transmitted to indicate if the bus is inverted. Several such coding techniques have been proposed to reduce bus activity using a lumped wire model without any inter-wire effects (e.g., capacitive coupling).

In this paper, we address a fundamentally different problem: the minimization of communication energy dissipation when the distributed and coupling effects of wires are considered. We propose here a comprehensive approach for evaluating the bus energy dissipation by assuming a coupled transmission line wire model. Using this model we conclude that minimizing the aver-

age numbers of transitions (i.e., the bus activity) is not necessarily the best approach for reducing energy. We observe that certain transitions are favored when energy is to be minimized. This has enabled us to develop an algorithm to tailor the transition profile over the data bus to minimize communication energy.

## 2. Energy Model with Interwire Parasitics

In this section we analyze the power consumption of the bus during a transition. We do this by taking into account the *distributed nature of the lines* and the *interwire parasitic capacitances*. We assume the following model for the lines and their drivers. For simplicity we first present the case for a two lines bus and later we give formulas for the general case.

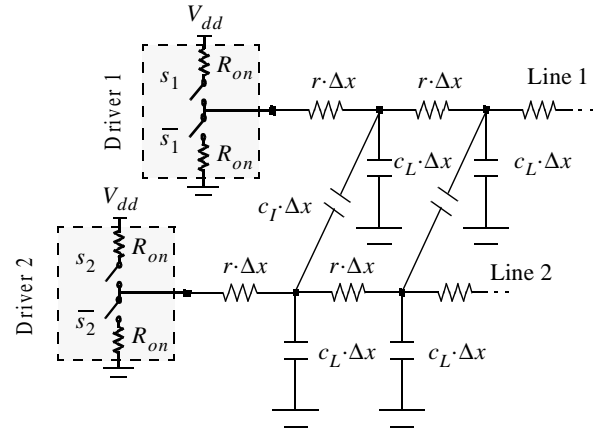


Figure 1: Lines and Drivers

Figure 1 shows the coupled transmission lines wire model. In this figure,  $r$  is the series resistance per unit length of the lines,  $c_L$  is the capacitance to ground per unit length and  $c_I$  is the inter-wire capacitance per unit length. (between adjacent lines). Let  $V_1^i, V_2^i$  be the initial voltages across the lines in Figure 1 (0 or  $V_{dd}$ ) and let  $V_1^f, V_2^f$  be the final voltages after the transition. We assume that from one transition to the next one there is enough time for the voltages of the lines to settle. We define the *lambda* ratio,  $\lambda \equiv c_I/c_L$  which is technology dependent. Let  $L_p$  be the physical length of the lines, then  $C_L = L_p \cdot c_L$  and  $C_I = L_p \cdot c_I$  are the total capacitances to ground and between

wires, respectively. For the rest of the paper we set  $C_L = 1$  and  $V_{dd} = 1$  in order to simplify the notation. To get the actual (physical) energy from the formulas below one should multiply the results by  $C_L \cdot V_{dd}^2$ . Table 1 shows the energy consumed by the drivers during the transition  $[V_1^i, V_2^i] \rightarrow [V_1^f, V_2^f]$  for the **two lines bus** of Figure 1 [14].

		$[V_1^f, V_2^f]$			
		00	01	10	11
$[V_1^i, V_2^i]$	00	0	$1 + \lambda$	$1 + \lambda$	2
	01	0	0	$1 + 2\lambda$	1
	10	0	$1 + 2\lambda$	0	1
	11	0	$\lambda$	$\lambda$	0

Table 1: **Energy Dissipation (normalized)**

Note that the energy loss caused by the interaction of the lines through  $c_l$  is captured by the  $\lambda$ 's in Table 1. By setting  $\lambda = 0$  we get the well known energy consumption associated to the activity of the lines. As technology ventures towards sub micron lengths,  $\lambda$  will increase thus making the energy lost through interwire capacitance significant. Therefore new coding schemes which use an interwire capacitance model should be designed, since current coding schemes may not be energy efficient.

Now we generalize the energy analysis for the case of busses with  $n$  lines. Let  $V^i = [V_1^i, V_2^i, \dots, V_n^i]^T$  be the initial voltages across the lines and  $V^f = [V_1^f, V_2^f, \dots, V_n^f]^T$  be the final ones after the transition. It can be shown that the energy consumed by the drivers during the transition  $V^i \rightarrow V^f$  is given by the formula [14],

$$E = (V^f)^T \cdot C_T \cdot (V^f - V^i) \quad (1)$$

where,

$$C_T = \begin{bmatrix} 1 + \lambda & -\lambda & 0 & \dots & 0 \\ -\lambda & 1 + 2\lambda & -\lambda & : & 0 \\ 0 & -\lambda & : & : & \\ : & : & : & 1 + 2\lambda & -\lambda \\ 0 & 0 & \dots & -\lambda & 1 + \lambda \end{bmatrix} \quad (2)$$

It is important to note that Equation 1 is still valid even if i) distributed parasitic self and mutual inductances among the lines are added and ii) parasitic lumped capacitors were added at the ends of the lines (loads / driving impedances).

## 2.1 A General Class of Coding Schemes

In this section, we describe a general class of bus coding schemes. Although many of the techniques proposed in the literature belong in this class, our unifying approach is new and results to exact closed form answers for the energy dissipation using coding, and power savings with respect to uncoded data transmission. In Section 2.4, an algorithm for finding efficient coding schemes is presented.

Figure 2 describes coding schemes having the property that the data,  $\{D(k)\}$  is encoded, transmitted and decoded “instantaneously”. In other words the data is available at the output of the receiver during the same period  $k$  it is fed into the encoder.

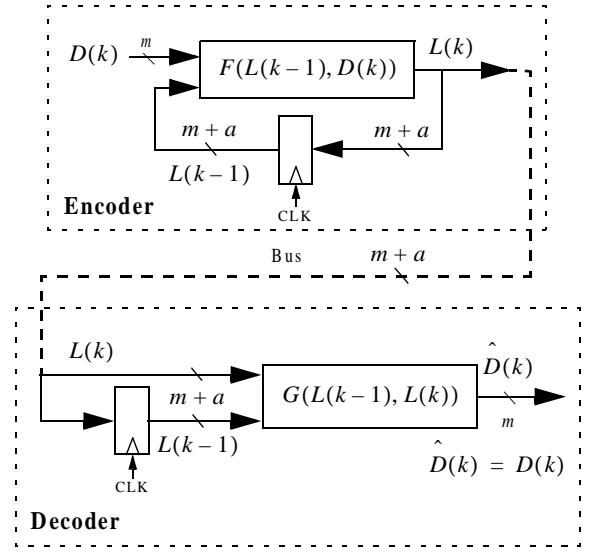


Figure 2: **The general class of Coding Schemes**

Note that the “original” data bus with  $m$  lines has now been expanded to a bus of  $m+a$  lines. We denote the *logical values* of these  $m+a$  lines by the vector  $L(k)$ . The vector  $L(k)$  is restricted to lay within the subset  $W = \{w_1, w_2, \dots, w_M\}$  of  $\{0, 1\}^{m+a}$  and we call the  $M$  elements of  $W$ , the *codewords* of the coding scheme.  $F$  and  $G$  are functions of the form,  $F : W \times \{0, 1\}^m \rightarrow W$  and  $G : W \times W \rightarrow \{0, 1\}^m$ . The decoder always recovers the original data i.e.  $\hat{D}(k) = D(k)$  if and only if the following condition holds,

$$G(w, F(w, d)) = d, \forall d \in \{0, 1\}^m, \forall w \in W \quad (3)$$

Relation (3) implies that for every fixed  $w \in W$  the mapping,  $d \rightarrow F(w, d)$  is injective. Even more, for every  $w$  the set  $X_w = \{F(w, d) : d \in \{0, 1\}^m\}$  that contains all the possible values of  $L(k)$ , given that  $L(k-1) = w$ , has exactly  $2^m$  elements.

We define the  $M \times M$  *transition matrix*  $T = [t_{i,j}]_{i,j=1}^M$  corresponding to the function  $F$  as,

$$t_{i,j} = \begin{cases} 1 & \text{if } w_j \in X_{w_i} \\ 0 & \text{otherwise} \end{cases}$$

Matrix  $T$  has exactly  $2^m$  ones in every one of its rows. We define the *transition graph*  $G_T$  which carries exactly the same information with the transition matrix as,

$$G_T = \{(w, F(w, d)) : w \in W, d \in \{0, 1\}^m\} \quad (4)$$

Relation (3) defines the restriction of the function  $G$  in the tran-

sition graph. The values of  $G$  in the set  $W \times W - G_T$  are immaterial and can be chosen in a convenient way to simplify the hardware implementation of the function.  $G_T$  has  $M \times 2^m$  elements. If the transition graph  $G_T$  has more than one strongly connected component [3], the coding scheme is degenerate in the sense that some of the codewords are not utilized. From now on we assume that  $G_T$  is *strongly connected*. Strong connectivity of  $G_T$  is equivalent to the *irreducibility* of the transition matrix  $T$  ([1],[2]), a property that will be used later.

## 2.2 A Motivational Example

Here is a simple TPC (transition pattern coding) where  $m = 2$ ,  $m + a = 3$ ,  $M = 6$ ,  $W = \{w_1, w_2, \dots, w_6\}$  and

$w_1 = 000$	$w_2 = 001$	$w_3 = 010$
$w_4 = 101$	$w_5 = 110$	$w_6 = 111$

The function  $F$  is given by the following diagram,

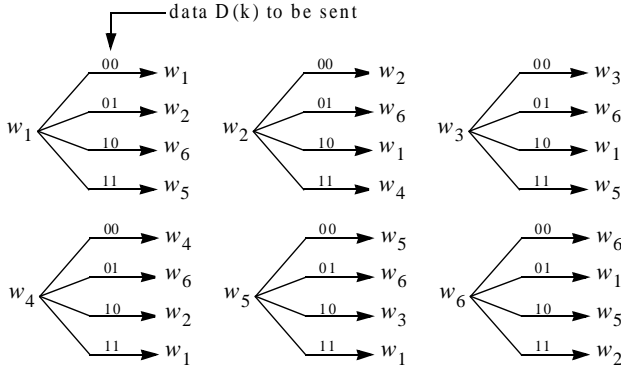


Figure 3: The transition graph of  $F$

and so the transition matrix is,  $T = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$ .

In this example, TPC favors transition patterns in which the voltages of neighboring bus lines change values in the same direction. This reduces the effective capacitance between adjacent wires. For a  $.18\mu m$  technology and minimum distance between the wires, the value of  $\lambda$  is about 3.2 and the energy saving of this coding scheme is about 17.9% even though the number of the bus lines has increased. This should not be surprising. In general the scheme encodes the information  $D(k)$  into transitions among codewords that cost less in average than the transitions between original data.

The following issues regarding TPC are presented in order in the next sections. i) **Exact calculation of the energy consumption.** ii) **Design of coding schemes.** iii) **Complexity reduction.**

## 2.3 Energy Consumption Using TPC

The energy consumption is related to the transition patterns among the codewords. Let  $\{D(k) = (d_1(k), d_2(k), \dots, d_m(k))^T\}_k$  be the input data sequence. We make the common assumption that the random variables  $d_j(k)$ ,  $j=1, \dots, m$ ,  $k=1, 2, \dots$  are *independent* and *uniformly distributed* in  $\{0, 1\}$ . This implies that the stochastic process  $L(k)$  (see Figure 2) is first order homogeneous Markov process [1] and the conditional probability for every possible transition among the codeword  $w_i \rightarrow w_j$  is  $P_r(L(k) = w_j | L(k-1) = w_i) = \frac{1}{2^m}$ . The *Probability Transition matrix*  $P$  of the coding block is defined as  $P = [P_{i,j}]_{i,j=1}^M = [P_r(L(k) = w_j | L(k-1) = w_i)]_{i,j=1}^M$ . It is,

$$P = \frac{1}{2^m} \cdot T \quad (5)$$

We define the *probability* (row) *vector*  $p(k)$  of the stochastic process  $L(k)$  as,  $p(k) = [P_r(L(k) = w_1), \dots, P_r(L(k) = w_M)]$ .

From the Chapman- Kolmogorov formula it is  $p(k) = p(0) \cdot P^k$  [1], where  $p(0)$  is the probability distribution of the initial state. The energy cost associated with a transition  $w_i \rightarrow w_j$  is denoted by  $C_t(w_i, w_j)$ . From Equation 1 we have that,

$$C_t(L(k-1), L(k)) = L(k)^T \cdot C_T \cdot [L(k) - L(k-1)]^T \quad (6)$$

Where the bits of the codewords  $L(k)$  are assumed to be zero or one in the field of real numbers and not in the Boolean algebra.

We call the matrix  $C = [C_t(w_i, w_j)]_{i,j=1}^M$  the *Cost Matrix* of the scheme. We define the *Time Averaged Expected Energy consumption* (TAAE) per transition of the bus  $E_a$ , as the limit of the average expected energy consumption during  $N$  consecutive transitions when  $N$  goes to infinity.

$$E_a = \lim_{N \rightarrow \infty} \frac{1}{N} \cdot \sum_{k=1}^N E[C_t(L(k-1), L(k))] \quad (7)$$

To get a compact expression for the TAAE we start from the

$$\text{relation } E[C_t(L(k-1), L(k))] = \sum_{i,j=1}^M P_{i,j} \cdot p_i(k-1) \cdot C_t(w_i, w_j)$$

where  $p_i(k)$  is the  $i$ -th entrance of the probability vector  $p(k)$  and  $P_{i,j}$  the  $i,j$ -th element of the transition probability matrix  $P$ . For matrices  $A, B$  of the same dimensions, let  $A \bullet B = [a_{i,j} \cdot b_{i,j}]_{i,j}$  be their Hadamard product and let  $\underline{1}$  be the (column) vector with all coordinates 1. Using the Chapman-Kolmogorov equations [1] we get,

$$E[C_t(L(k-1), L(k))] = p(0) \cdot P^{k-1} \cdot (P \bullet C) \cdot \underline{1} \quad (8)$$

Now recall our assumption of the *strong connectivity* of the transition graph  $G_T$  or equivalently the *irreducibility* of the transi-

tion probability matrix  $P$ . Even more  $P$  is a row stochastic matrix by definition. We need the following modified version of the Perron-Frobenius theorem. (corollary 8.4.6 in [1]).

### Theorem 1 (Perron-Frobenius)

An irreducible row stochastic matrix  $P$  is always similar to a matrix of the form,

$$\Delta = \begin{bmatrix} \Delta_1 & 0 \\ 0 & J \end{bmatrix} \quad (9)$$

where  $J$  is a matrix in Jordan form with eigenvalues of modulus less than one and  $\Delta_1$  is the following diagonal matrix.

$$\Delta_1 = \text{Diag} \left( 1, e^{\frac{2\pi i}{q}}, e^{\frac{4\pi i}{q}}, \dots, e^{\frac{2(q-1)\pi i}{q}} \right) \quad (10)$$

$q$ , is the number of eigenvalues of  $P$  of modulus one and is always equal or greater than one. •

Definition 7, relations (5),(8) and the Perron-Frobenius theorem lead to the following formula for the Time Averaged Expected Energy,

$$E_a = \frac{1}{2^m} \cdot b^T \cdot (T \cdot C) \cdot \underline{1} \quad (11)$$

$b$  is the left eigenvector of matrix  $P$  corresponding to eigenvalue one and i.e.  $b^T \cdot P = b^T$  (or equivalently  $b^T \cdot T = 2^m \cdot b^T$ ) and satisfying  $b^T \cdot \underline{1} = \underline{1}$ .

## 2.4 Algorithm for Deriving Coding Schemes

An algorithm for deriving efficient coding schemes is presented in this section. Given the parameters  $m, \alpha$  (see Figure 2) and  $\lambda$  the algorithm targets to the coding scheme with the lowest energy cost (TAEE). It searches among coding schemes with  $W = \{0, 1\}^{m+a}$  and  $M = 2^{m+a}$ . Although it is heuristic, the algorithm gives Transition Pattern Coding schemes with significant energy savings.

The intuition behind its recursion is that if  $T(k) = T$  for all iterations, then the entry  $\phi_{i,j}$  of the matrix  $\Phi = [\phi_{i,j}]_{i,j=1}^M$  at the  $k$ -th iteration equals the expected cost of a sequence of  $k$  transitions starting with  $w_i \rightarrow w_j$ . Even more,  $\xi_i$  at the  $k$ -th iteration is the expected cost of a sequence of  $k$  transitions starting from state  $w_i$ . In every iteration the value of matrix  $T = T(k)$  is selected such that the values of  $\xi_i$  for all  $i = 1, \dots, M$  are minimized.

Begin

$$M := 2^{m+a}$$

$$N := 2^m$$

$$W = \{w_1, \dots, w_M\} = \{0, 1\}^{m+a}$$

$$C := [C_t(w_i, w_j)]_{i,j=1}^M$$

$$\xi = (\xi_1, \dots, \xi_M)^T = (0, 0, \dots, 0)^T$$

For  $k = 1$  to  $2M$  do

$$\Phi := C + \frac{1}{2} \cdot \xi^T$$

For  $i = 1$  to  $M$  do

find  $1 \leq j_1 < j_2 < \dots < j_N \leq M$  such that

$$\sum_{r=1}^N \phi_{i,j_r} = \min_{1 \leq \eta_1 < \eta_2 < \dots < \eta_N \leq N} \sum_{r=1}^N \phi_{i,\eta_r}$$

For  $i = 1$  to  $M$  do

$$t_{i,p} := \begin{cases} 1 & \text{if } p \in \{j_1, j_2, \dots, j_N\} \\ 0 & \text{otherwise} \end{cases}$$

End

$$\xi_i := \frac{1}{N} \cdot \sum_{r=1}^N \phi_{i,j_r}$$

End

$$T(k) := [t_{i,j}]_{i,j=1}^N$$

End

End

## 2.5 Results of the Algorithm

For  $\lambda$  varying from 0 to 10,  $m = 2, 3, 4, 5$  and some values of  $a$  from 1 to 5, a TPC coding scheme was derived using the algorithm above. Its energy saving was calculated (exactly) and compared with that of the Bus Invert coding scheme. The results are shown in Figure 4.

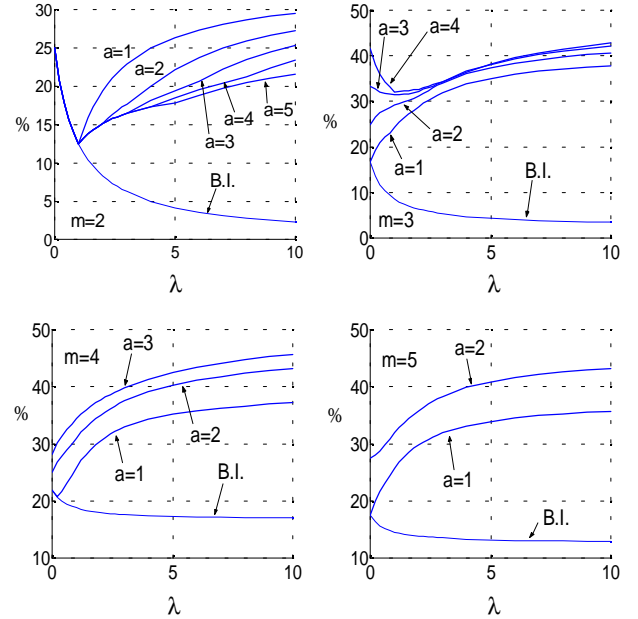


Figure 4: Energy Savings of TPC and Bus Invert

Generally the efficiency of the T.P.C. increases with  $\lambda$ . It can become more than 50% as  $\lambda$  approaches infinity. The typical values of  $\lambda$  for  $.18\mu\text{m}$  technology and minimal distances between the wires are higher than 3.

## 2.6 Copying With Complexity

A way to reduce complexity is to split the data bits into groups and then encode each group independently of the others. The approach is shown below.

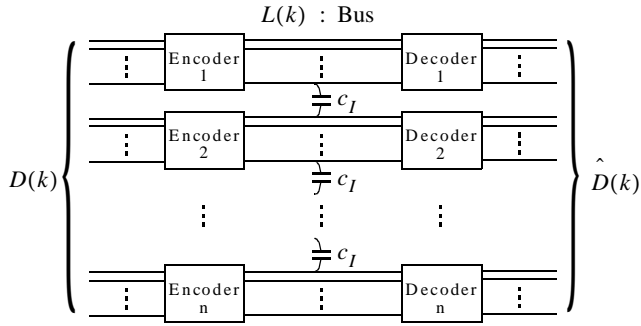


Figure 5: **Partitioned Coding Scheme:**  
**A juxtaposition of simpler blocks**

The energy dissipation in the partitioned coding scheme equals the sum of the energy dissipations of the individual blocks plus the energy losses due to the interactions between the adjacent blocks. This interactions take place between the last line of the 1st block and the first line of the 2nd block, the last line of the 2nd block and the first line of the 3rd etc. The calculation of the expected energy dissipation caused by the interactions is presented in the next section.

## 2.7 The Interaction Energy

The energy loss caused by the interaction of two consecutive blocks corresponds to the  $\lambda$ 's of Table 1 if  $V_1, V_2$  correspond to the touching boundary lines of the two blocks. For simplicity, all coding blocks are assumed similar with parameters  $m, a, W, M, F, G$  as defined in Section 2.1. Let  $L(k)$  and  $L'(k)$  be the codewords of two consecutive coding blocks  $B$  and  $B'$ . Suppose  $B$  is above  $B'$  (see Figure 5). Let  $l(k)$  be the *last* (bottom) bit of  $L(k)$  and  $l'(k)$  be the *first* (top) bit of  $L'(k)$ . So  $l(k)$  and  $l'(k)$  correspond to adjacent lines in the bus. From Table 1 we extract the Table 2 for the interaction energy at time  $k$ . Call this energy cost,  $J(k)$

$J(k)$		$[l(k+1), l'(k+1)]$			
		00	01	10	11
$[l(k), l'(k)]$	00	0	$\lambda$	$\lambda$	0
	01	0	0	$2\lambda$	0
	10	0	$2\lambda$	0	0
	11	0	$\lambda$	$\lambda$	0

Table 2: **Energy Loss caused by Interaction**

The random variables  $l(k)$  and  $l'(k)$  are independent (because the data bits are independent) and the expected value of  $J(k)$  is given, after some minor manipulation, by,

$$E[J(k)] = \lambda \cdot P_r(l(k+1) = 0) \cdot P_r(l'(k) = 0, l'(k+1) = 1) +$$

$$\begin{aligned} & \lambda \cdot P_r(l(k) = 1, l(k+1) = 0) \cdot P_r(l'(k+1) = 1) + \\ & \lambda \cdot P_r(l(k) = 0, l(k+1) = 1) \cdot P_r(l'(k+1) = 0) + \\ & \lambda \cdot P_r(l(k+1) = 1) \cdot P_r(l'(k) = 1, l'(k+1) = 0) \end{aligned} \quad (12)$$

Let  $W_{0*}$  and  $W_{1*}$  be the subsets of  $W$  containing all codewords whose *first* bit is 0 and 1 respectively. Similarly let  $W_{*0}$  and  $W_{*1}$  be the subsets of  $W$  containing all codewords whose *last* bit is 0 and 1 respectively. Then, for any  $\alpha, \beta$  in  $\{0, 1\}$  it is,

$$\begin{aligned} & P_r(l(k) = \alpha, l(k+1) = \beta) = \\ & = \sum_{v \in W_{*\alpha}, w \in W_{*\beta}} P_r(L(k) = v, L(k+1) = w) \\ & = \sum_{v \in W_{*\alpha}, w \in W_{*\beta}} P_r(L(k+1) = w | L(k) = v) \cdot P_r(L(k) = v) \end{aligned}$$

Similarly it is,  $P_r(l'(k) = \alpha, l'(k+1) = \beta) =$

$$\sum_{v \in W_{\alpha*}, w \in W_{\beta*}} P_r(L'(k+1) = w | L'(k) = v) \cdot P_r(L'(k) = v)$$

To get a compact expression for  $E[J(k)]$  we need a few more definitions. For  $\alpha = 0, 1$  and  $i = 1, \dots, M$  let,

$$h_{*\alpha}^i = \begin{cases} 1 & \text{if } w_i \in W_{*\alpha} \\ 0 & \text{if } w_i \notin W_{*\alpha} \end{cases}, \quad h_{\alpha*}^i = \begin{cases} 1 & \text{if } w_i \in W_{\alpha*} \\ 0 & \text{if } w_i \notin W_{\alpha*} \end{cases}$$

We define the four diagonal matrices,

$$\begin{aligned} H_{*\alpha} &= \text{diag}(h_{*\alpha}^1, h_{*\alpha}^2, \dots, h_{*\alpha}^M) \\ H_{\alpha*} &= \text{diag}(h_{\alpha*}^1, h_{\alpha*}^2, \dots, h_{\alpha*}^M) \end{aligned}$$

Then, using the above definitions we can write that,

$$P_r(l(k) = \alpha, l(k+1) = \beta) = p(k) \cdot H_{*\alpha} \cdot P \cdot H_{*\beta} \cdot \underline{1} \quad (13)$$

$$P_r(l'(k) = \alpha, l'(k+1) = \beta) = p(k) \cdot H_{\alpha*} \cdot P \cdot H_{\beta*} \cdot \underline{1} \quad (14)$$

and that,

$$P_r(l(k+1) = \alpha) = p(k+1) \cdot H_{*\alpha} \cdot \underline{1} \quad (15)$$

$$P_r(l'(k+1) = \alpha) = p(k+1) \cdot H_{\alpha*} \cdot \underline{1} \quad (16)$$

Combining Equations (12)-(16) and using the Chapman-Kolmogorov formula [1] we get expression (17) for the expected value of the interaction energy.

$$\begin{aligned} & E[J(k)] = \\ & = \lambda \cdot [p(0) \cdot P^{k+1} \cdot H_{*0} \cdot \underline{1}] \cdot [p(0) \cdot P^k \cdot H_{0*} \cdot P \cdot H_{1*} \cdot \underline{1}] \\ & + \lambda \cdot [p(0) \cdot P^{k+1} \cdot H_{1*} \cdot \underline{1}] \cdot [p(0) \cdot P^k \cdot H_{*1} \cdot P \cdot H_{*0} \cdot \underline{1}] \\ & + \lambda \cdot [p(0) \cdot P^{k+1} \cdot H_{0*} \cdot \underline{1}] \cdot [p(0) \cdot P^k \cdot H_{*0} \cdot P \cdot H_{*1} \cdot \underline{1}] \\ & + \lambda \cdot [p(0) \cdot P^{k+1} \cdot H_{*1} \cdot \underline{1}] \cdot [p(0) \cdot P^k \cdot H_{1*} \cdot P \cdot H_{0*} \cdot \underline{1}] \end{aligned} \quad (17)$$

We define the *Time Averaged Expected Energy Consumption due to Interaction* (TAEEI) as,

$$E_{ai} = \lim_{N \rightarrow \infty} \frac{1}{N} \cdot \sum_{k=1}^N E[J(k)] \quad (18)$$

From theorem 1 the matrix  $P$  can be written in the form,

$$P = A \cdot \Delta \cdot B \quad (19)$$

Where  $\Delta$  is as in theorem 1,  $A = [\underline{1}, a_1, \dots, a_{M-1}]$  consists of the (right) eigenvectors of  $P$  appropriately ordered and  $B = [b_0, \dots, b_{M-1}]^T$  is the inverse of  $A$ . Using (19) we get that,

$$E_{ai} = \lambda \cdot b_0^T \cdot [H_{1*} \cdot P \cdot H_{0*} + H_{*1} \cdot P \cdot H_{*0}] \cdot \underline{1} \\ q^{-1} \\ - 4 \cdot \sum_{r=1}^{q-1} \sin^2\left(\frac{\pi \cdot r}{q}\right) \cdot |p(0) \cdot a_r|^2 \cdot (b_r^T \cdot H_{*0} \cdot \underline{1}) \cdot (\bar{b}_r^T \cdot H_{0*} \cdot \underline{1}) \quad (20)$$

Where  $\bar{b}$  is the conjugate vector of  $b$  and  $q$  is given in theorem 1. In the (most common) case  $q = 1$ , formula (20) simplifies to,

$$E_{ai} = \frac{\lambda}{2^m} \cdot b_0^T \cdot [H_{1*} \cdot T \cdot H_{0*} + H_{*1} \cdot T \cdot H_{*0}] \cdot \underline{1} \quad (21)$$

## 2.8 Total Energy Consumption of the Partitioned Coding Scheme

Having formulas for the *Time Averaged Expected Energy consumption* of the individual coding blocks and also formulas the *Time Averaged Expected Energy consumption due to Interaction* between the blocks, the *Total Time Averaged Expected Energy consumption* (TTAEE) of the whole coding scheme in Figure 5 is given by (22).

$$E_T = n \cdot E_a + (n-1) \cdot E_{ai} \quad (22)$$

where  $n$  is the number of blocks. For the case  $q = 1$  we have,

$$E_T = \frac{n}{2^m} \cdot b_0^T \cdot (T \cdot C) \cdot \underline{1} \\ + \frac{\lambda \cdot (n-1)}{2^m} \cdot b_0^T \cdot [H_{1*} \cdot T \cdot H_{0*} + H_{*1} \cdot T \cdot H_{*0}] \cdot \underline{1} \quad (23)$$

Note that this energy is normalized with respect to  $V_{dd}$  and  $C_L$ , so the actual energy consumed is  $V_{dd}^2 \times C_L \times E_T$ .

Figure 6 presents the energy saving of the TPC given by the algorithm for some combinations of the parameters, *number of blocks*  $n$ , *data lines*  $m$ , *bus lines*  $m+a$ , *lambda*  $\lambda$ .

## 3. Conclusions

Minimizing transition activity is not necessarily the best approach to reduce energy dissipation when the effect of inter-wire capacitance is significant. An accurate energy model has enabled the development of efficient Transition Pattern Coding strategies using an elaborate distributed model for the wires. An efficient heuristic algorithm was developed to design the coding strategies. The overall energy dissipation can be reduced by a factor of 2.

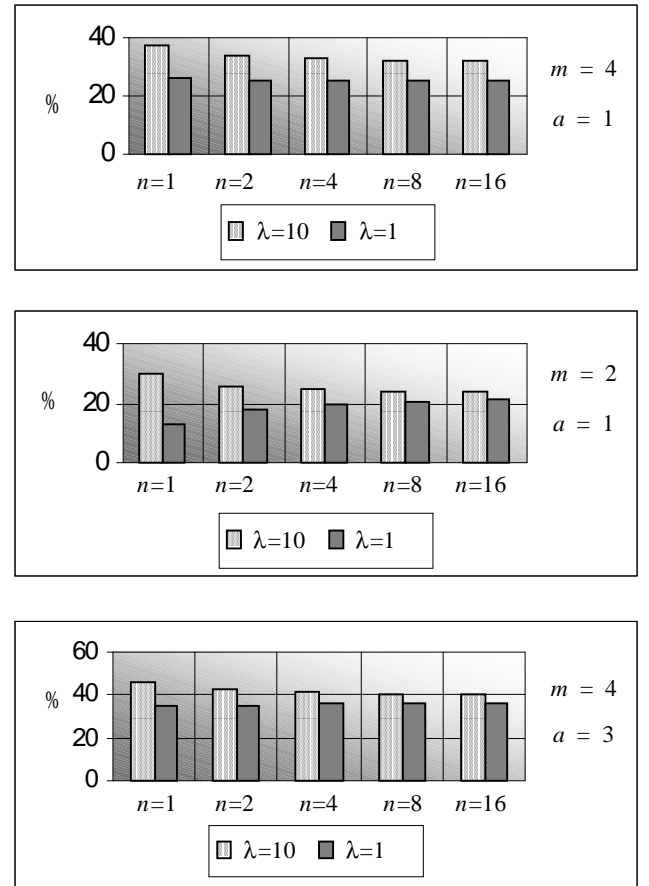


Figure 6: Percentage of Energy Saving using Partitioned Coding Schemes

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## References

- [1] Robert G. Gallager, "Discrete Stochastic Processes", Kluwer Academic Publishers 1998.
- [2] Roger A. Horn and Charles R. Johnson, "Matrix Analysis", Cambridge University Press 1994.
- [3] Frank Harary, "Structural models: an introduction to the theory of directed graphs", New York, Wiley 1965.
- [4] A. Chandrakasan, R. Brodersen, "Low power CMOS Design", IEEE Press, 1998.
- [5] K.Y. Khoo, A. Willson, Jr., "Charge recovery on a databus," IEEE/ACM International Symposium on Low Power Electronics and Design, pp. 185-189, 1995.
- [6] B. Bishop, M. J. Irwin, "Databus charge recovery: Practical consideration," International Symposium on Low Power Electronics and Design, pp. 85-87, August 1999.

- [7] Y. Nakagome, K. Itoh, M. Isoda, K. Takeuchi, M. Aoki, "Sub-1-V swing internal bus architecture for future low-power ULSI's," *IEEE Journal of Solid-State Circuits*, pp. 414-419, April 1993.
- [8] M. Hiraki, H. Kojima, H. Misawa, T. Akazawa, Y. Hatano, "Data-dependent logic swing internal bus architecture for ultralow-power LSI's," *IEEE Journal of Solid-State Circuits*, pp. 397-401, April. 1995.
- [9] H. Yamauchi, H. Akamatsu, T. Fujita, "An asymptotically zero power charge-recycling bus architecture for battery-operated ultrahigh data rate ULSI's," *IEEE Journal of Solid-State Circuits*, pp. 423-431, April. 1995.
- [10] H. Zhang, J. Rabaey, "Low swing interconnect interface circuits," *IEEE/ACM International Symposium on Low Power Electronics and Design*, pp. 161-166, August 1998.
- [11] M. Stan, W. Burleson, "Low-power encodings for global communication in cmos VLSI," *IEEE Transactions on VLSI Systems*, pp. 49-58, Vol. 5, No. 4, Dec. 1997.
- [12] S. Ramprasad, N. Shanbhag, I. Hajj, "A coding framework for low-power address and data busses," *IEEE Transactions on VLSI Systems*, pp. 212-221, Vol. 7, No. 2, June 1999.
- [13] J.M. Rabaey, *Digital Integrated circuits*. Prentice Hall 1996.
- [14] P. Sotiriadis, A. Wang, A. Chandrakasan, "Transition Pattern Coding: An approach to reduce Energy in Interconnect", *ESSCIRC' 2000*, Stockholm, Sweden, Sept. 2000.