On-chip picosecond time measurement

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Abstract— **A flash Time to Digital Converter can be calibrated to precision on the order of the arbiter aperature without precise input signals. A theoretical result useful for calibration of a noise-limited arbiter array is derived, and verified empirically. A test chip with 64 arbiters in a 0.35 μm CMOS process shows temporal resolution better than 2 picoseconds.**

I. Introduction and Background

While increasing resources are devoted to implementing low skew and low jitter clocks in modern microprocessors, there are few ways to measure jitter. Skew can be measured by such methods as e-beam and photonic emission, but because both average thousands of edges, neither method is suitable for resolving cycle-by-cycle clock jitter. The circuit and method presented here were developed to measure jitter between clock domains, but apply directly to measuring the timing of any signal relative to a reference.

Given enough time, the output of an arbiter, or synchronizer, will settle to either a logic '1' or '0'. The output, y, is not deterministic; $y(t) = 1$ if and only if $t > t_{os} + t_n$, where t is the time difference between the two inputs, t_{os} is the input-referred time offset due to mismatch and t_n is noise with standard deviation σ [1], [2]. The strong sensitivity of y to t near $t = t_{os}$ makes the arbiter useful for precise time measurement.

II. Time-to-digital converter

A flash converter is characterized by its comparison thresholds, collectively labeled x. In a flash Time-Digital Converter (TDC), the x are offset times to which the input t is compared. x could be set by a delay vernier, as in Fig. 1 (*cf.* the resistor string in a flash ADC). D would be either explicit delays, or differences in t_{os} of the arbiters. Variation of t_{os} — the standard deviation is about $15ps$ in $0.35\mu m$ CMOS — adds directly to x and thus limits the precision. Fig. 2.a shows a plot of ideal x for an 8-level converter; Fig. 2.b shows the actual positions of the x with normally distributed t_{os}.

Fortunately, it is possible to measure the arbiter offsets and thus calibrate a TDC very accurately; given enough arbiters, the precision is limited by calibration, not by manufacturing tolerances. The precision limit for a fixed-size converter is achieved by removing the explicit vernier delays entirely; $x_i = t_{os,i}$ will suffice to distribute x. Fig. 2.c shows typical x for such a converter.

III. Calibration

The TDC could be calibrated directly by connecting two signals with precisely-known t and measuring resulting outputs for t over the range of interest. Unfortunately, input jitter adds linearly to the apparent measurement noise in this case. In cases where it is impossible or inconvenient to input known signals, it is also possible to calibrate a flash TDC indirectly with *uncorrelated* signals. For uniformly distributed t, the probability that t is measured between two sampling thresholds, $P(x_i + t_n > t > x_j + t_n) \triangleq P_{ij}(01)$, is proportional to $x_i - x_j \triangleq \Delta_{ij}$ for a single event, as long as the difference is much larger than sampling noise, $\Delta_{ij} \gg t_n$. For example, if the two input signals are constant-frequency square waves, measurements with bit i low and bit j high will occur with a frequency of $\Delta_{ij} f_1 f_2$ where f_1 and f_2 are the frequencies of the two input signals. While x can be fully deduced from such measurements, the resolution is poor for $\Delta_{ij} \approx t_n$.

A second indirect calibration method resolves small Δ_{ij} in terms of σ. When Δ_{ij} is comparable to t_n, there will sometimes be a "bubble" in the output codeword; that is, it will appear that $x_j + t_n > t > x_i + t_n$ even though $x_i > x_j$. The ratio $r = P_{ij}(10)/P_{ij}(01)$ depends only on $\delta = \Delta_{ij}/\sigma$, as derived in *Derivation*, below. Thus, by measuring r and inverting Eq. 8, one can find relative spacings of x in terms of σ. Combined with either of the previous two methods calibrations, this measurement thus gives σ and precise measurements of x. Note that both indirect methods are completely insensitive to input jitter. In this way an array of arbiters can be calibrated to much higher precision than their manufacturing tolerances without the use of precise input clocks.

IV. Circuit and Results

The circuit is a direct copy of a flash analog to digital converter (ADC) structure, with comparators replaced by arbiters as shown in Fig. 1. A symmetric CMOS arbiter is shown in Fig. 3. For the test chip, 64 such arbiters were connected in parallel to two test inputs, and their outputs individually recorded.

Fig. 4 shows x for one test chip measured directly. As expected, process variations distribute the x over a range of approximately 50 picoseconds. A plot of x calculated by numerically inverting Eq. 8 for measured data vs. x measured directly is shown in Fig. 5. The fit is perfect to within the tolerances of the measurement equipment; clearly, calibration by random signals is viable. Best fit σ is 0.35 picoseconds, which corresponds to an arbiter aperature of $\approx 1ps$, consistent with a previously reported simulated value of 10ps in a $3\mu m$ CMOS process. Nonuniform spacing of the arbiter thresholds limits resolution of this TDC to 2ps over the range [-15ps,15ps].

References

[1] Linsay Kleeman. The jitter model for metastability and its application to redudnant synchronizers. *IEEE Transactions on Computers*, 39(7):"930–942", July 1990.

[2] W. A. M. Van Noije, W. T. Liu, and Jr. J. Navarro S. Precise final state determination in mismatched CMOS latches. *Journal of Solid State Circuits*, 30(5):"607–611", May 1995.

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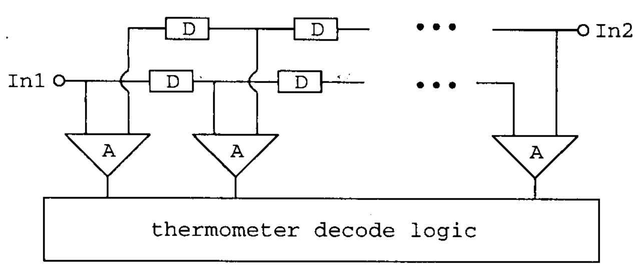

Fig. 1. TDC structure. "D" marks delay elements, and "A" the arbiters.

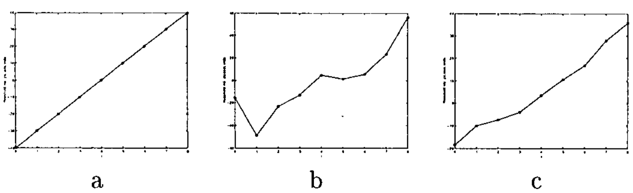

Fig. 2. $x(i)$ vs. i (a) Ideal, $x_i \propto i$ (b) $x_i \propto i + t_{os}$, $15ps$ std. dev. (c) $x_i = t_{os}$, $15ps$ std. dev., sorted by x_i.

Derivation

Consider two arbiters with $t_1 = x_1 + t_{n1}$ and $t_2 = x_2 + t_{n2}$. t_1 and t_2 are the instantaneous switching thresholds of the arbiters, so

$$P(y_1 = 1) = P(t > t_1) \tag{1}$$

$$P(y_2 = 0) = P(t < t_2) \tag{2}$$

$$P(y_1 = 1, y_2 = 0) \triangleq P_{12}(10) = P(t_1 < t < t_2) \tag{3}$$

$$P_{12}(10) = P(t_1 < t_2) \cdot P(t_1 < t < t_2 \,|\, t_1 < t_2) \tag{4}$$

Let $x = t_2 - t_1$. Then x is Gaussian with mean $x_2 - x_1 = \Delta_t$ and standard deviation 2σ. For uniformly distributed t, $P(t_1 < t < t_2 \,|\, t_1 < t_2) \propto t_2 - t_1$. Substituting into Eq. 4,

$$P_{12}(10) \quad \propto \quad x \cdot P(x > 0) \tag{5}$$

$$\propto \quad \int_0^\infty x \frac{1}{\sqrt{4\pi}\sigma} e^{\dfrac{-(x-\Delta_t)^2}{4\sigma^2}} dx \tag{6}$$

$$\propto \quad \frac{\sigma}{\sqrt{2\pi}} e^{\left(\dfrac{-\Delta_t^2}{4\sigma^2}\right)} + \frac{\Delta_t}{2}\left(1 + \operatorname{erf}\left(\frac{\Delta_t}{2\sigma}\right)\right) \tag{7}$$

By symmetry, $P_{12}(01)|_{[\sigma,\Delta_t} = P_{12}(10)|_{\sigma,-\Delta_t}$. Defining $\delta = \frac{\Delta_t}{2\sigma}$ and $\operatorname{erfcx}(x) = e^{x^2} \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$

$$r(\delta) \triangleq \frac{P_{12}(10)}{P_{12}(01)} = \frac{1 + \sqrt{\pi}\delta \cdot \operatorname{erfcx}(-\delta)}{1 - \sqrt{\pi}\delta \cdot \operatorname{erfcx}(\delta)} \tag{8}$$

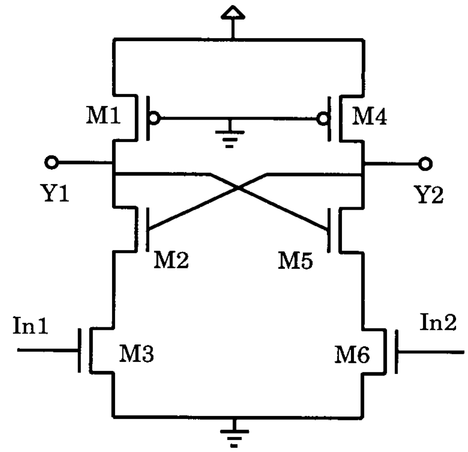

Fig. 3. Symmetric CMOS arbiter

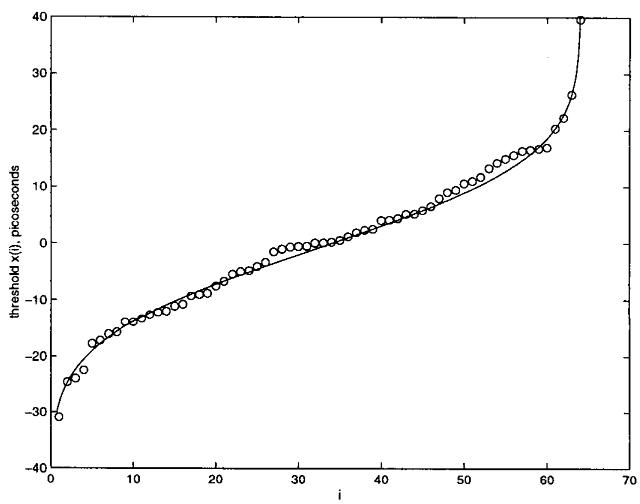

Fig. 4. Measured x_i, with expected curve for 18ps standard deviation of t_{os}.

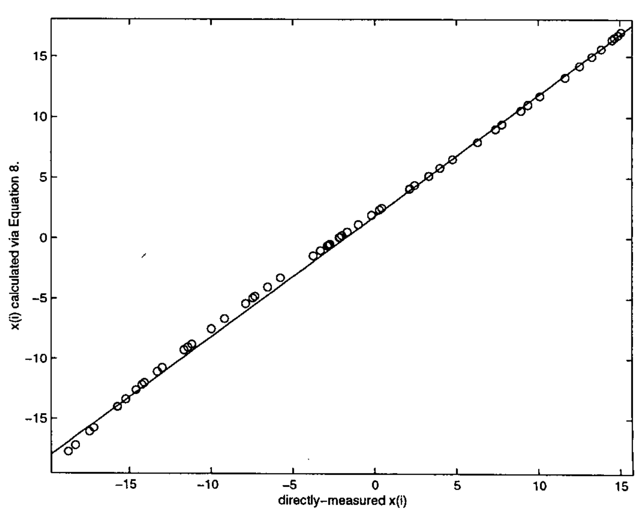

Fig. 5. Measured x_i vs. x_i derived via Eq. 8, for $\sigma = 0.35ps$