Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science
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6.050J/2.110J **Information and Entropy** **Spring 2006**

Issued: **February 7, 2006** **Problem Set 1-2 Solutions** **Due: February 10, 2006**

Solution to Problem 1: No-NOR Workaround

We are asked to check the equality between the circuit in Figure 1–1 and the NOR gate.

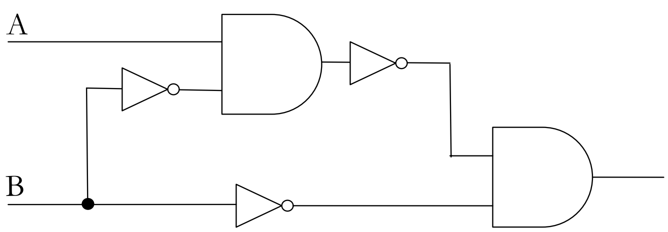

Figure 1–1: Logic circuit

Solution to Problem 1, part a.

Here we present two ways to test this identity: direct algebraic manipulation and the truth tables.

Algebraic Manipulation

First we notice that the circuit in Figure 1–1 can be written as an algebraic expression:

$$\overline{B} \cdot \overline{(A \cdot \overline{B})}. \tag{1–1}$$

Using de Morgan's Law on the parenthesized expression:

$$\overline{B} \cdot (\overline{A} + B). \tag{1–2}$$

Applying the distributive property for the OR:

$$(\overline{B} \cdot B) + (\overline{A} \cdot \overline{B}). \tag{1–3}$$

The first parenthesized expression is always false, and can thus be ignored. We recognize de Morgan's Law again in the second parenthesized expression:

$$(\overline{A} \cdot \overline{B}) = \overline{A + B} \tag{1–4}$$

and the right hand side is simply the NOR gate. Other proofs exist that use the unnamed theorem.

A	B	$\overline{B}$	$A \cdot \overline{B}$	$\overline{A \cdot \overline{B}}$	$\overline{B} \cdot (\overline{A \cdot \overline{B}})$	NOR
0	0	1	0	1	1	1
0	1	0	0	1	0	0
1	0	1	1	0	0	0
1	1	0	0	1	0	0

Table 1–1: Truth table analysis of Problem 1

Truth Table Analysis

We can also prove this identity by writing out the truth tables. By hand it looks like this, with all intermediate calculations:

Solution to Problem 1, part b.

We are interested in reducing the number of gates in the control circuit from Figure 1–1. We can, and have already shown so in the last step from our proof, see equation (1–4). The left hand side of (1–4) translates to the circuit in figure 1–2. We note that this is the same conclusion we would have reached by simple application of De Morgan's law to the NOR gate.

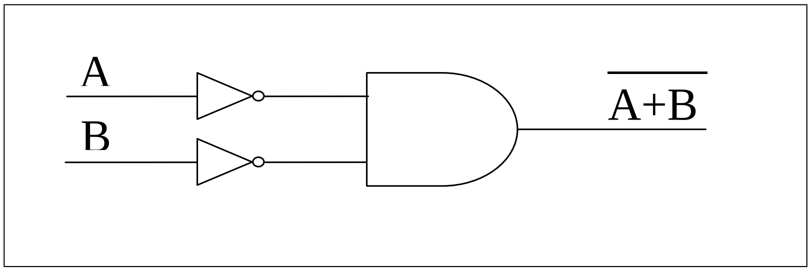

Figure 1–2: Logic circuit

Solution to Problem 2: How Many Bits ... ?

Solution to Problem 2, part a.

In order to assign unique barcodes to all the particles, we need 10^{90} numbers that can be represented at least with $\log_2(10^{90}) = 90 \log_2(10)$ bits. Note that from $10^3 \approx 2^{10}$, we also have $\log_2(10) \approx \frac{10}{3}$. Therfore, the barcode has about 300 bits.

Solution to Problem 2, part b.

- There are 10^{10} possible values for the result of the lengh measurement, so it provides $\log_2(10^{10}) = 10 \log_2(10) \approx 33.3$ bits of information.
- accordingly, there are $10^{10} \times 10^{10} \times 10^{10} = 10^{30}$ possibilities for the location of the molecule in the room; therefore, giving $30 \log_2(10) \approx 100$ bits of information.

Solution to Problem 2, part c.

- Each of the 10^8 spaces can independently take any of the 2^7 possible characters. The number of possible ararngement of these characters and consequently the number of books in the library of Babel is $\underbrace{2^7 \times 2^7 \times \ldots \times 2^7}_{10^8 \text{times}} = (2^7)^{10^8}$. Note that one of the characters being NULL, we have already accounted for the books with less number of characters.

- As any possible arrangement of 10^8 characters would be a different book, in order to have a unique title or catalog card entry for each book, we need at least the same number of 10^8 characters.

- If we assume the same 10^8 character limit for the characters of all books there, we have the same number of $(2^7)^{10^8}$ possible arrangements of characters in any of them. Thus, each book might have approximately $\log_2((2^7)^{10^8}) = 7 \times 10^8$ bits or almost 100 Mega bytes. As there are about 29 million books in the Library of Congress (http://www.loc.gov/homepage/fascinate.html,) there would be $29 \times 10^6 \times 7 \times 10^8 \approx 2 \times 10^{14}$ bits.

Solution to Problem 2, part d.

According to ITU (International Telecommunication Union), in 2003 there were about 593,000,000 personal computers all around the world. If we assume an average of 10 Giga bytes or equialently 8×10^9 bits of information in each PC, there would be around 6×10^{19} bits in all PCs.

Solution to Problem 2, part e.

- Human brain has about 100 billion (10^{11}) neurons, each of which has an average 7,000 synaptic connections to other neurons in the brain (http://en.wikipedia.org/wiki/Neuron.)

  These 10^{11} neurons result in around 37 bits required to label each neuron. We would say the total amount of information is equal to the number of synapse:

  $$10^{11}\,\text{neurons} \times 7000\,\text{synapses/neuron} = 7 \times 10^{14}\,\text{synapses}$$

  times the amount of information per synapse which is the number of bits required to label destination neuron (37) plus one bit to label degree and type of excitation or inhibition of synapse. So, the total information is around 3×10^{16} bits. This should be right within an order of magnitude or so: it's the information required to wire up a brain or an artificial model of the brain. A few more bits might be added to classify different types of neurons or differentiate more precisely among different levels of excitation but this should be close.

- The mass of an adult brain is about 1,300 to 1,400 grams. As 78% of the brain is composed of water, let us estimate the number of atoms in the brain with the assumption that it is completely made of water. Therefore, human brain would contain $\frac{1{,}400(\text{g})}{18(\text{g/mol})}$ moles and as we know each mole has 6.02×10^{23} molecules. In case of water, each molecule has 3 atoms; thus, brain would have about 1.4×10^{26} atoms.