| 6.050J/2.110J | Information and Entropy | Spring 2006 |

Issued: February 6, 2006   **Problem Set 1-2**   Due: February 10, 2006

Problem 1: No-NOR Workaround

You are finishing up your robot control system late one night and run out of NOR gates. The stockroom is closed for the evening, and none of your friends has any NOR gates. You only have AND and NOT gates available.

The 6.050/2.110 TA suggests that you implement NOR with AND and NOT gates using a circuit like the one on figure 1–1.
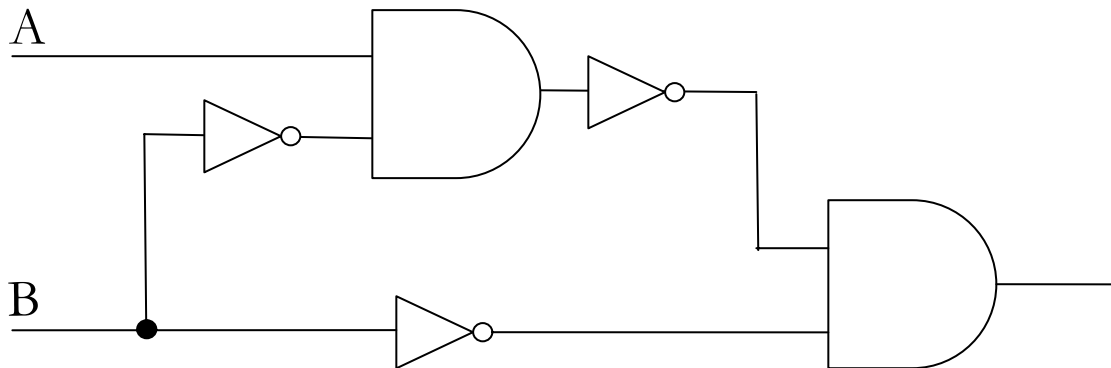

Figure 1–1: Logic circuit

You wonder if this circuit is actually equivalent to the NOR function, so you decide to find out.

a. Using the truth table of the circuit or by any other means, test this expression for all possible values of A and B, and see whether it is equivalent to a NOR gate.

b. You have always been suspicious of the smartness of your TA; therfore, you wonder if this is really a good circuit for what you want to do. In particular, you want to know if there is a simpler circuit (for example with fewer number of gates) that can realize the NOR gate with ANDs and NOTs. Either give the Boolean expression for such a circuit, or write a paragraph justifying the need for all the used gates.

Problem 2: How Many Bits ... ?

In this problem you will estimate the potential amount of the information content of different physical systems.

    a. There are approximately 10^{90} elementary particles in the universe. If we were to assign each of them a unique barcode, how many bits would that barcode have? Where would you put the barcode?

    b.
- A typical atom is about 10^{-10} meters across (10^{-10} meters = 1 Angstrom). If a meter stick can resolve distance to the level of a single atom, how many bits of information does it provide?
- How many bits are required to describe the position of a single molecule in a 10 m by 10 m by 3 m room if the minimum length scale is 10^{-10} meters?

    c.
- In Jorge Luis Borges' story, *The Library of Babel*, a library contains books containing every possible sequence of characters. Assuming each character is labelled in ASCII code by 7 bits, and books are limited to no more than 10^8 characters, how many books are there in the library?
- In the previous part, how long is the card catalog entry for each book? If each book has a unique title, how long is its title? How much would you be willing to pay for a library card in the library of Babel?
- How many bits are there in all the books in the Library of Congress? (Feel free to use the web to figure out how many books there are.)

    d. Estimate how many bits there are in all the computers in the world.

    e.
- Give a rough estimate of how many bits of information your brain can store. First, find the average number of neurons in the brain. Then, find the number of synapses per neuron (a synapse is a connector between neurons; for our purposes, it can be either excitatory or inhibitory, so each synapse requires one bit to characterize). Then estimate the number of ways of wiring the neurons together. For this purpose, you may assume that each neuron could be connected to any other neuron in the brain; this assumption is definitely false, but it will tend to overestimate the number of possible connections, so you will get an upper limit to the number of bits.
- How many bits could your brain store if every atom in your brain could contain one bit?