

## Quiz Solutions

### Solution to **Problem 1: (24%)**

Solution to Problem 1, part a.

$$P(C = 0) = \underline{3/4} \quad P(C = 1) = \underline{1/4}$$

Solution to Problem 1, part b.

$$I = \underline{2 \text{ bits}} \quad J = \underline{0.82 \text{ bits}} \quad L = \underline{1.18 \text{ bits}} \quad N = \underline{0 \text{ bits}} \quad M = \underline{0.82 \text{ bits}}$$

Solution to Problem 1, part c.

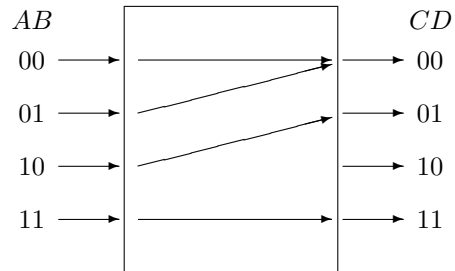


Figure Q-1: Ben's gate

Solution to Problem 1, part d.

$$P(CD = 00) = \underline{1/2} \quad P(CD = 01) = \underline{1/4} \quad P(CD = 10) = \underline{0} \quad P(CD = 11) = \underline{1/4}$$

Solution to Problem 1, part e.

$$I = \underline{2 \text{ bits}} \quad J = \underline{1.5 \text{ bits}} \quad L = \underline{0.5 \text{ bits}} \quad N = \underline{0 \text{ bit}} \quad M = \underline{1.5 \text{ bits}}$$

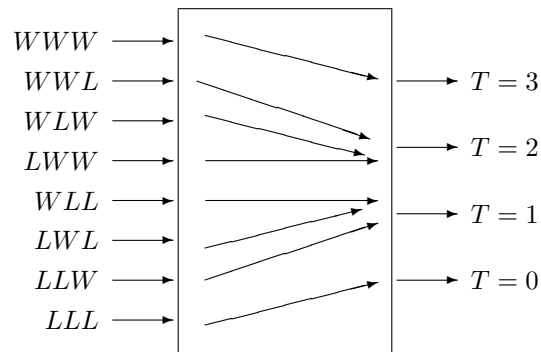
Solution to Problem 1, part f.

No, this gate has loss so it is not reversible.

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**Solution to Problem 2: (35%)**

Solution to Problem 2, part a.



$$P(T = 0) = \underline{0.125} \quad P(T = 1) = \underline{0.375} \quad P(T = 2) = \underline{0.375} \quad P(T = 3) = \underline{0.125}$$

Solution to Problem 2, part b.

$$I_{in} = \underline{3 \text{ bits}} \quad I_{out} = \underline{1.81 \text{ bits}} \quad L = \underline{1.19 \text{ bits}} \quad N = \underline{0} \quad M = \underline{1.81 \text{ bits}}$$

Solution to Problem 2, part c.

Code:

$$T = 0 : \underline{000} \quad T = 1 : \underline{01} \quad T = 2 : \underline{1} \quad T = 3 : \underline{001}$$

$$\text{Average message length in bits: } \underline{1.875 \text{ bits}} \quad \text{Average message cost in dollars: } \underline{23.75 \text{ cents}}$$

**Solution to Problem 3: (40%)**

Solution to Problem 3, part a.

$$P \underline{1/3} \quad N \underline{1/3} \quad Q \underline{1/3}$$

Solution to Problem 3, part b.

$$\text{Constraint Equation: } \underline{P + 5N + 25Q = 4}$$

Solution to Problem 3, part c.

$$P_{min} \underline{0.25} \quad P_{max} \underline{0.875} \quad N_{min} \underline{0} \quad N_{max} \underline{0.75} \quad Q_{min} \underline{0} \quad Q_{max} \underline{0.125}$$

**Solution to Problem 3, part d.**

Write this formula as a function of any one of the three probabilities  $C$ ,  $E$ , and  $R$ .

$$S = P \log_2 \left( \frac{1}{P} \right) + N \log_2 \left( \frac{1}{N} \right) + Q \log_2 \left( \frac{1}{Q} \right) \quad (\text{Q-1})$$

For example in terms of  $Q$ :

Equation for the Entropy:

$$\frac{-Q \log_2 Q - (0.75 - 6Q) \log_2 (0.75 - 6Q) - (0.25 + 5Q) \log_2 (0.25 + 5Q)}{\quad}$$

**Solution to Problem 3, part e.**

$$P \underline{0.875} \quad N \underline{0} \quad Q \underline{0.125} \quad \text{Entropy} = \underline{0.544}$$

**Solution to Problem 3, part f.**

Not a good guess since we have just found a set of probabilities with higher entropy. The guess therefore cannot be the maximum entropy.