Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
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6.050J/2.110J **Information and Entropy** **Spring 2005**

Issued: March 7, 2005

Problem Set 6

Due: March 11, 2005

Problem 1: When Hell Freezes Over...

Last week, we introduced you to Bayes' Theorem, and you used it to track characteristics through one generation of common garden peas. Since the theorem is so important, we'd like you to apply it in a new context. You may have wondered why the sentence "Tech is Hell" is so popular. Bayes's rule can help us there.

Say, there is a 15% chance that MIT will close (call this event C) given that Hell freezes over (call this event F). Indeed Hell always freezes over on days that MIT is closed. Overall, the probability that Hell freezes over any particular day is 1%.

In solving this problem, use Bayes's Theorem and the notation for probability introduced last week. For this purpose you may neglect leap years, so every year has exactly 365 days. Please do not round your answers.

a. What is the probability of MIT closing on any given day, and how would you represent it symbolically?

b. What is the probability of MIT closing and Hell freezing over on the same day, and how is it represented symbolically?

c. How many days a year is Hell expected to freeze over based on the above?

d. How many days a year is MIT expected to close based on the above?

e. This year MIT closed during a snow emergency. What is the probability that it will close again next year?

Problem 2: Communicate

a. . A very noisy symmetric binary channel is able to transmit 1000 bits per second, but has a 25% probability of introducing an error into each bit transmitted. What is its channel capacity in bits per second?

One application proposed for this channel requires an error rate of less than 10% but the bit rate need not be as large as 1000 bits per second. Consider counteracting the noise by using triple redundancy error correction. Define a new channel, consisting of the old channel with the attached coder and decoder.

b. What is the bit rate of this newly defined channel?

c. What is its error rate?

d. What is its channel capacity?

e. Will the suggested solution meet the needs of the application? at what cost?

Problem 3: Nerds' Nest Expands Operations

Remember the trendy new cafe on campus for which you designed an error-correcting code? As more and more patrons trust the reliability of the cafe, the manager has started to think about expanding the menu to include two entrees.

The manager admires the gadgets in the Kendall-MIT T stop, and believes that a similar system to place orders for entrees will capture the attention of customers as much as these gadgets amuse the T patrons.

He has devised the following communication system to pass orders to the kitchen

- Customers choose their entree. (Since the system is experimental, they are only allowed to choose one entree at a time). The entree choices are "Mahi Mahi" (F) and "Huge Ravioli" (R). The Marketing expert has anticipated that 80% of the customers will prefer "Mahi Mahi" over the "Ravioli".
- Customers have a knob and a lever. They choose an entree, set the knob accordingly, and push the lever a few times, setting in motion a hammer that will hit one of two gongs heard in th ekitchen – low pitch gong for Ravioli, and high pitch for Mahi Mahi.
- Upon hearing the tone, the cooks (really, cuisine-consultants) will simply determine the cooking conditions and, when the food is ready, will pass it to the conveyor belt that will take it to the custommer.

This extravagant comunication system can process one order per second, exactly the same rate at which customers arrive. The manager is convinced that the uniqueness of the system will generate excitement and is already dreaming of franchises all over campus. Nonetheless, the manager has some doubts because he fears that the system may not work if customers send orders at exactly the rate at which they can be accepted. And that's why he seeks once more your advice.

As you inspect the system, you realize that the gong mechanism is not always reliable; often the hammer hits the gong too soft for the cook to be able to differentiate the high pitch sound, and hears a low pitch sound instead. The result is that a 50% of the times that Mahi Mahi is ordered, the cook hears the low-pitch gong. On the other hand, every time Ravioli is ordered no confusion happens.

a. Denote the perception of low-pitch gong by the cooks as outcome L and perception of high-pitch gong as H. Viewing the lever-gong system as a communications channel with noise, give the transition probability matrix with elements c_{LR}, c_{LF}, c_{HR}, c_{HF}.

b. What are the probabilities of outcomes L and H?

c. What is the uncertainty about the entree selected right before the customer activates the lever?

d. Find the channel capacity C_{ideal} of this channel if there were no noise. (Hint: note that it is a binary channel, but not a symmetric one.)

e. What is the mutual information between input and output?

f. What is the maximum mutual information over all possible input probability distributions?

g. What is the Channel capacity of this channel?

Turning in Your Solutions

If you used MATLAB for this problem set, you may have some M-files and a diary. You may turn in this problem set by e-mailing your M-files and diary to 6.050-submit@mit.edu. Do this either by attaching them to the e-mail as *text* files, or by pasting their content directly into the body of the e-mail (if you do the latter, please indicate clearly where each file begins and ends). Alternatively, you may turn in your solutions on paper in room 38-344. The deadline for submission is the same no matter which option you choose.

Your solutions are due 5:00 PM on Friday, March 11, 2005. Later that day, solutions will be posted on the course website.