Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
Department of Mechanical Engineering

6.050J/2.110J **Information and Entropy** **Spring 2005**

Issued: May 3, 2005

Problem Set 12

Due: May 7, 2005

Problem 1: The beaver cooler

The heat engine described in the Chapter 12 is just one type of Heat Machine. The underlying mechanism to exchange work and heat catches your fancy because it seems to be versatile. You have read that the same type of cycle can help you build heat pumps and refrigerators, it all depends on what system you are interested in and so, you decide to make a refrigerated cage for your pet, a beaver. You figure that you can make the device work as a refrigerator, transferring energy from a lower temperature cage to your higher temperature living room. The cycle of the heat engine is pictured here (the cycle of your refrigerator may be different but has the same rectangular shape).

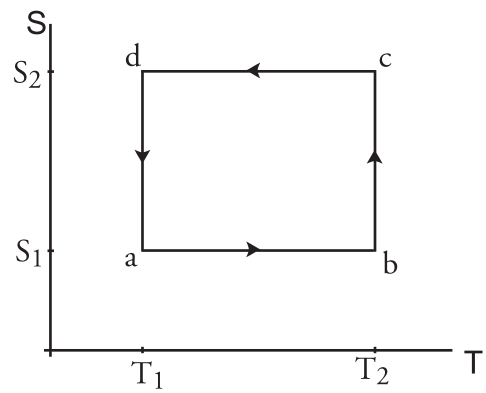

Figure 12–1: Heat Engine Cycle

You have devised a way to make a large number of dipoles behave in unison, like a single dipole with a much large magnetic moment. Looking in your basement, you find you have 10^6 dipoles at your disposal, each with a dipole moment of a single electron, 1.165×10^{29} Joule-meters/Ampere. Thus there are two states, with energies

State	Energy
up	$-m_{eff}H$
down	$m_{eff}H$

Table 12–1: System Parameters

where $m_{eff} = 1.165 \times 10^{-23}$ Joule-meters/ampere is the effective dipole moment of this array.

In a refrigerator the idea is to take heat from the low-temperature (T_1) environment (in this case the beaver cage) into the working material (in this case the magnetic dipole) in order to heat the material inside the high-temperature (T_2) environment. You decide to run the system using a rectangular path similar to the one pictured above.

a. To use the device as a refrigerator, you can either run it as shown above or in the reverse direction. Which should you choose? Hint: a reversible heat engine, when run backwards, can serve as either a refrigerator or a heat pump.

Your living room is at room temperature (17 degrees Celsius). You want your beaver cage to be at a temperature slightly above freezing ($5°C$).

b. Express the room and cage temperatures in Kelvin.

Continuing your design, you want to calculate how much energy can be extracted from the cage and thrown into the living room. You figure that an inexpensive permanent magnet could be moved toward and away from the array of dipoles, and its field at the dipoles could be made as large as 1000 amperes/meter. You decide to run the cycle as follows:

Point	a	b	c	d
H (A/m)	?	1000	500	?

Table 12–2: More Parameters

Now you want to calculate the work and heat into the system during each of the four legs of the cycle. On each leg, the change in energy is related to the work and heat as $dE = dw + dq$. According to Chapter 12, the work and heat are related to changes dH and dS as $dw = (E/H)\,dH$ and $dq = TdS$. The way the cycle is defined it is relatively easy to find dq since on each leg either S or T is constant.

c. In order to analyze this cycle you must first determine the magnetic field H_a and H_d at points a and d. Focus first on the leg c to d, where $dS = 0$ and $dT = T_2 - T_1$. This equation from Chapter 12 is helpful:

$$TdS = \left(\sum_i p_i(E_i(H) - E)^2\right) \frac{1}{k_B T} \left(\frac{1}{T}dT - \frac{1}{H}dH\right) \tag{12–1}$$

Use this equation to determine a relationship among T_1, T_2, H_c, and H_d.

d. From this relationship find H_d.

The "coefficient of performance" of a refrigerator is defined as the ratio of heat extracted from the cold environment to the work done on the system, in our case by the magnetic field. The larger this coefficient, the less power is needed for the same amount of cooling. Surprisingly, the coefficient of performance can be found without actually solving for the amount of energy converted. The coefficient is therefore universal, not depending on the details of the refrigerator but only on the two temperatures T_1 and T_2.

e. Find the coefficient of performance of this refrigerator.

f. Now focus on the leg b to a. Determine a relationship among T_1, T_2, H_a, and H_b.

g. Calculate the magnetic field H_a.

h. Next you want to know the work and heat into the system during the four legs of the cycle. First consider the top leg, d to c. What is the heat into the system dq on that leg?

To go further you have to calculate the probabilities, since you need them to find the energy E at each of the four corners. You already know the temperature and magnetic field at each corner, so it is straightforward to find α and then the probabilities using these equations from Chapter 12:

$$p_i = e^{-\alpha} e^{-E_i/k_B T} \tag{12–2}$$

$$\alpha = \ln\left(\sum_i e^{-E_i/k_B T}\right) \tag{12–3}$$

i. Find the two probabilities p_{up} and p_{down} at each of the four corners of the cycle. Note that for adiabatic legs, the probabilities do not change so for example they are the same at points a and b.

j. Find the energy $E = \sum_i E_i p_i$ for each of the four corners.

k. Find the difference between the two entropies, $S_2 - S_1$.

l. Find the heat dq for each of the four legs (you have already found one of these). Hint: the heat is zero for two of the legs.

m. Find the work dw for each of the four legs by calculating the change in energy and subtracting the heat.

n. Find the net work into the system in one cycle, starting at point a and ending up at the same point. Hint: it is positive, meaning that energy actually is put in as work.

o. Calculate the ratio of heat out at the lower temperature T_1 to work in during one cycle. Compare this to the coefficient of performance calculated earlier.

p. Your beaver likes his peanut butter cool. How many cycles would this machine require to cool down one gram of peanut butter one degree Celsius? (Assume that the specific heat of peanut butter is approximately 2 Joules per degree Celsius per gram.)

Problem 2: Information is Cool

An exciting application of thermodynamics is in the study of biological systems. The energy cycles that we have seen in class are the starting point for modelling processes that occur throughout nature, whether it be the dynamics of a muscle cell or an animal's thermal regulation facilities. For this problem we will examine forms of energy loss and storage in the human body. Please note that the unit *Calorie*, used to indicate the energy content of foods you consume, is actually a *kilocalorie* which is equal to 4.1868×10^3 Joules. The approximate basal metabolic rate of an average 70 kg adult male is 75 Calories/hour (that is, the body dissipates 75 Calories per hour to maintain its normal body temperature while performing normal bodily tasks). What a person doesn't take in through diet will ultimately be burned from the body's store of fat, and when a person takes in more Calories than is needed the excess energy is normally stored as fat. One kilogram of fat corresponds to 33.1×10^6 Joules of metabolized energy.

Have fun with these simple exercises in energy conversion.

a. Express the basal metabolic rate, 75 Calories/hour, in Watts, or Joules per second. Optional question: You know what a 100-Watt light bulb looks like when turned on. Why don't people look like that?. Consider a Glowworm, and assume that the mechanism to produce light is similar to that of a 30Watt lightbulb. How much heat would a glowworm have to dissipate in 1 hour? What would be its difference in temperature with the environment after 1 hour from lighting up if its specific heat is 1 Cal/g/K and its weight 100g?

b. How many Calories will a person who does no exercise (75 Calories/hr) need to eat in one day to maintain normal body weight? The brain is said to be responsible of 20% of that energy consumption; how many of those Calories are devoted to thinking?

c. A person who jogs normally burns 830, rather than 75, Calories/hour. A swimmer at a moderate pace burns 696 Calories in the same time. Consider three people who follow exactly the same routine and eat the same food for one year, except that Seth does no exercise, Luis swims three times per week for thirty minutes, and Paul jogs for the same time that Luis swims, and after each jog he replenishes himself by consuming a serving of his favorite fluid, Ben and Jerry's Cherry Garcia ice cream. According to the label, a serving of this ice cream contains 260 Calories. If Luis neither gains nor loses weight during this year, how much weight do Seth and Paul gain or lose? If Paul does not want to gain or loose weight, how much ice-cream should he eat?

d. In the cold winter of Cambridge, heat from dorm rooms is lost to the outdoors through the windows at 2.052×10^8 J/hour. MIT administration has known for quite a while that on average, MIT undergraduate students dissipate heat at 70Watts when sleeping, 100 Watts when awake and 250 Watts when doing problem sets, and it takes them about 2 hours per problem set. If on average there are 300 students in the dorm, how many problem sets should MIT recommend the undergraduate students do to compensate for the energy loss? *Optional:* Use the principle of maximum entropy to determine the proportion of undergraduate students that are sleeping, awake, and solving problem sets, assuming that you are told that heat loss is compensated.

e. Paul drinks his root beer cold (0 degrees Celsius), arguing that a significant portion of the Calories consumed are used to raise the temperature of the beverage to body temperature (98.6 degrees Fahrenheit), rather than going into fat. Are you convinced? Consider that a 330 ml can of A&W root beer has 170 Calories, and that about one Calorie is needed to raise one liter of water one degree Celsius.

Turning in Your Solutions

You may turn in this problem set by e-mailing your written solutions, M-files, and diary to 6.050-submit@mit.edu. Do this either by attaching them to the e-mail as text files, or by pasting their content directly into the body of the e-mail (if you do the latter, please indicate clearly where each file begins and ends). If you have figures or diagrams you may include them as graphics files (GIF, JPG or PDF preferred) attached to your email. Alternatively, you may turn in your solutions on paper in room 38-344. The deadline for submission is the same no matter which option you choose.

Your solutions are due 5:00 PM on Friday, May 7, 2005. Later that day, solutions will be posted on the course website.