Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
Department of Mechanical Engineering

6.050J/2.110J **Information and Entropy** **Spring 2005**

Issued: April 25, 2005

Problem Set 11

Due: April 29, 2005

Problem 1: Entropy Goes Up

This problem is based on the magnetic dipole model in Chapter 11 of the notes. The system is assumed to have one dipole, as pictured, and for the purposes of this problem there is only one environment and it also contains exactly one dipole (this is to keep the calculations simple). For this problem you may assume that the environment and system have the same applied magnetic field.

The configuration is set up with the system having a high energy and the environment a low energy, and then the two are allowed to interact with the result that some energy in the form of heat may flow from the environment to the system. You will calculate the amount of heat and the entropy before and after this operation.

The configuration is shown in Figure 11–1.

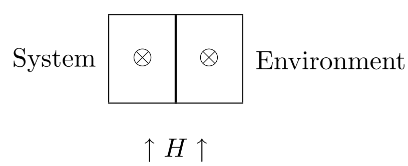

Figure 11–1: Dipole moment example.

Each of the dipoles shown can be either aligned with the field, in which case it contributes an energy $-m_d H$ or in the opposite direction, in which case it contributes $m_d H$. Thus the system has two states, one with energy $m_d H$, and one with energy $-m_d H$. The environment also has two states, with the same energies. As the problem starts, the two (system and environment) are isolated and each has a probability distribution as shown in these tables.

System				Environment			
State	Dipole	Energy $E_{s,i}(H)$	Probability $p_{s,i}$	State	Dipole	Energy $E_{e,j}(H)$	Probability $p_{e,j}$
$i = 0$	up	$-m_d H$	0.4	$j = 0$	up	$-m_d H$	2/3
$i = 1$	down	$m_d H$	0.6	$j = 1$	down	$m_d H$	1/3

Table 11–1: System and Environment Parameters

a. Find the system energy E_s (the expected value of the energy). You may leave your answer (and the other energies asked for) as a multiple of $m_d H$.

b. Find the environment energy E_e.

c. Find the environment entropy S_e. You may leave your answer (and the other entropies asked for) in terms of k_B.

d. Find the system entropy S_s.

e. Recall from the notes the units of k_B and entropy. What are the units of β and α?

Now consider what happens when the barrier between the system and the environment is removed, so they can interact. After some time has passed, your knowledge of the initial probabilities and separate energies is no longer relevant, and you can only treat the system as a whole. The Principle of Maximum Entropy can be used to estimate the probabilities of the four possible states (up-up, up-down, down-up, and down-down) without your assuming any information you do not have.

f. What is the total energy E_t?

g. Find β_t.

h. Find the four probabilities $p_{t,i,j}$.

i. Calculate the total entropy S_t and compare it to the sum of the system and environment entropies before the interaction.

j. Calculate the expected value of energy E_s in the system. (*Hint: You may find useful to look at this problem as you did for inference. After mixing, the information about the distribution of states in the system was lost. So it is difficult to compute the expected value of energy in the system, yet you have the information about the distribution of states in the total system, and using inference,... .*)

k. How much energy came into the system from the environment in the form of heat during the interaction?

l. What is the change in entropy of the system after mixing with the environment?.

Problem 2: Conveyor Belt Power

Belt Power Inc. has developed a new technology to charge PDA and mobile devices in airport conveyor belts and they are currently lobbying to get the system adopted. The idea is to convert part of the mechanical energy of conveyor belts to electricity and have users charge their PDAs using plugs conveniently located in the hand rails. The airport authority, clearly out of its domain here, seeks your advice to analyze the design and determine if it will work, and if so how much energy can be converted from mechanical to electrical form. (Although this problem does not involve either information or entropy, it has the same approach to energy conversion that will be used in describing heat engines and the second law of thermodynamics.)

The device uses several metal plates hidden in the machinery of the conveyor belt. As the conveyor belt turns, the metal plates in its upper and lower surfaces cross each other and simulate a capacitor with a changing area. Figure 11–1 shows the laboratory prototype with a fixed lower plate and a moving upper plate.

One of the plates is wired to a three-way switch (see the circuit in Figure 11–1) that can connect it to a low-voltage battery with voltage V_{low} volts, a high-voltage battery with voltage V_{high} volts, or neither.

Your knowledge of electrostatics enables you to deduce the following.

- When the plates are partially aligned, the charge q is related to voltage v between the plates by the "parallel-plate capacitance" formula $q = \epsilon_0 Av/d$ where ϵ_0 is the permittivity of free space, 8.854×10^{-12} farads per meter, A is the area of overlap, and d is the distance between plates.
- Perfect alignment of the plates (shown in Figure 11–1) occurs when $A = A_{max}$, and as the overlap decreases the charge decreases as well.

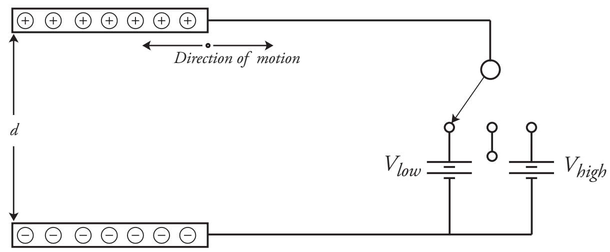

Figure 11–1: Charge pump

- The charge q can only change if the switch connects the top plate with one of the batteries, in which case the energy supplied by the battery to change the charge from q_1 to q_2 is $v(q_2 - q_1)$.

The designer of the prototype describes the operation of charging the high-voltage battery as follows.

1. Start with the low-voltage battery connected and the electrodes in alignment, i.e., $A = A_{max}$.
2. Throw the switch to the middle position so the charge cannot change (the engineer knows how to design a circuit to throw the switch automatically in the final product).
3. As the upper plate moves (in either direction) the electrodes pull apart. Since the charge cannot change, the voltage rises. Continue until the voltage is equal to v_{high}.
4. Throw the switch so that the high-voltage battery is connected. Since the voltage is equal to the voltage of that battery this operation does not by itself affect q.
5. Let the upper plate continue to move until there is virtually no overlap ($A = A_{min}$). This causes the charge q to decrease, meaning charge is being delivered to the high-voltage battery.
6. Throw the switch to the middle position.
7. Let the upper plate move so that the overlap of the electrodes increases. Since the charge cannot change, the voltage must go down. Continue until the voltage reaches v_{low}.
8. Throw the switch to connect to the low-voltage battery.
9. Let the plate move until the electrodes are once more in alignment.
10. The plate position and the charge are what they were at the beginning of these instructions, and some charge has been placed on the high-voltage battery thereby charging it up, and the same amount of charge has been taken from the low-voltage battery, thereby discharging it.
11. Repeat steps 2 - 9 as required to obtain the desired battery charge.

Naturally you want to know how much energy has been added to the high-voltage battery and how much was lost from the low-voltage battery, and therefore how much was supplied by the motion.

a. To start your analysis, plot the charge for a given voltage as a function of plate position, between $A = A_{min}$ and $A = A_{max}$. (take $A_{min} = 0.1cm^2$)

b. Draw the charging cycle as a rectangle in the charge-voltage plane (charge on the vertical axis, voltage on the horizontal).

c. Mark the parts of this diagram that correspond to the numbers in the description above.

d. Indicate for each of the four legs whether mechanical energy is being supplied to the device or taken from the device, and whether electrical energy is being supplied to or taken from either of the batteries.

e. Find a formula for the charge q_0 delivered to the high-voltage battery in one cycle as a function of A_{min}, A_{max}, d, v_{low}, v_{high}, and ϵ_0.

f. Find the energy supplied to the high-voltage battery per cycle (this is the product of its voltage times the charge supplied).

g. Find the energy delivered by the low-voltage battery per cycle.

h. Find the energy supplied by the mechanical source per cycle.

i. By taking the device apart you discover that the plates are square 10 cm on a side, the minimum overlap between plates is $A = 0.1cm^2$, and the gap d is 2mm (the plates have some flexibility so that the conveyor belt can fold). You want to charge a 12V storage battery using a 1.5V battery for the low voltage. How long would it take to charge it with 10^{-9} coulombs (enough to run 1 nanowatt load for 9 seconds)? The typical sped of a conveyor belt is 75 cm/s. How long must the conveyor belt be?

Turning in Your Solutions

You may turn in this problem set by e-mailing your written solutions, M-files, and diary to 6.050-submit@mit.edu. Do this either by attaching them to the e-mail as text files, or by pasting their content directly into the body of the e-mail (if you do the latter, please indicate clearly where each file begins and ends). If you have figures or diagrams you may include them as graphics files (GIF, JPG or PDF preferred) attached to your email. Alternatively, you may turn in your solutions on paper in room 38-344. The deadline for submission is the same no matter which option you choose.

Your solutions are due 5:00 PM on Friday, April 29, 2005. Later that day, solutions will be posted on the course website.