Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
Department of Mechanical Engineering

6.050J/2.110J **Information and Entropy** **Spring 2005**

Issued:

Problem Set 10 Solutions

Due:

Solution to Problem 1: Well, Well, Well

Solution to Problem 1, part a.

Inside the well V(x) = 0 and therefore

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} \tag{10–6}$$

Solution to Problem 1, part b.

If E has a nonzero imaginary part E_{imag}, then the magnitude of $f(t)$ is a function of time, in particular

$$\mid f(t) \mid = exp(E_{imag}t/\hbar) \tag{10–7}$$

If $E_{imag} > 0$ then $\mid f(t) \mid$ gets large for large values of t (i.e., it blows up at infinity). If $E_{imag} < 0$ then $\mid f(t) \mid$ gets large for large values of $-t$ (i.e., it blows up at negative infinity). In either case it is impossible to normalize $\psi(x)$.

Solution to Problem 1, part c.

$$E\phi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2\phi(x,t)}{\partial x^2} \tag{10–8}$$

Solution to Problem 1, part d.

Since

$$\begin{aligned}
\phi(x) &= a\sin(kx) + b\cos(kx) && (10–9)\\
\frac{d\phi(x)}{dx} &= ak\cos(kx) - bk\sin(kx) && (10–10)\\
\frac{d^2\phi(x)}{dx^2} &= -ak^2\sin(kx) - bk^2\cos(kx)\\
&= -k^2\phi(x) && (10–11)
\end{aligned}$$

Therefore

$$E\phi(x) = \left(\frac{\hbar^2k^2}{2m}\right)\phi(x) \tag{10–12}$$

so

$$E = \frac{\hbar^2k^2}{2m} \tag{10–13}$$

Solution to Problem 1, part e.

One of the boundary conditions is $\phi(0) = 0$, so

$$\begin{aligned} 0 &= \phi(0) \\ &= asin(0) + bcos(0) \end{aligned} \tag{10–14}$$

The other boundary condition is $\phi(L) = 0$, so

$$\begin{aligned} 0 &= \phi(L) \\ &= asin(kL) + bcos(kL) \end{aligned} \tag{10–15}$$

From the first equation, we determine that b has to be zero, since $cos(0) = 1$ and the equality would otherwise not hold. The value of a can then be determined normalizing $\phi(x)$.

Solution to Problem 1, part f.

$\phi(x)$ must be zero at the boundaries, which implies

$$sin(kL) = 0 \Rightarrow kL = \frac{\pi}{2}2jk \qquad = \frac{j\pi}{L} \tag{10–16}$$

Solution to Problem 1, part g.

$$e_j = \frac{\hbar^2 \pi^2 j^2}{2mL^2} \tag{10–17}$$

Solution to Problem 1, part h.

$$\phi_j(x) = a \sin\left(\frac{j\pi x}{L}\right) \tag{10–18}$$

Solution to Problem 1, part i.

$$e_1 = \frac{\hbar^2 \pi^2}{2mL^2} \tag{10–19}$$

Solution to Problem 1, part j.

$$e_2 = \frac{\hbar^2 \pi^2}{mL^2} \tag{10–20}$$

Solution to Problem 1, part k.

$$e_1 = \frac{\hbar^2 \pi^2}{2mL^2} \tag{10–21}$$

$$= \frac{(1.054 \times 10^{-34}\text{Joule-seconds})^2 \times (3.1416)^2}{2 \times (9.109 \times 10^{-31}\text{kilograms}) \times (1.5 \times 10^{-9}\text{meters})^2} \tag{10–22}$$

$$= 2.71 \times 10^{-22}\text{Joules} \tag{10–23}$$

(10–24)

Solution to Problem 1, part l.

Express this ground-state energy in electron-volts (1 eV= 1.602×10^{-19} Joules).

$$\begin{aligned} e_1 &= 3.765 \times 10^{-23}\text{Joules} \\ &= 1.692 \times 10^{-3}\text{eV} \end{aligned} \tag{10–25}$$