Basic Set Notation for Events

Intersection

$A \cap B$ = all outcomes that lie in event A and event B

Example

Outcomes are selection of a single MIT student.

Events are sets of students (freshmen, females, undergrads, and their intersections, unions and complements)

grad students $\cap$ women = all female grad students

grad students $\cap$ freshmen = ϕ, the empty set

Union

$A \cup B$ = all outcomes for which event A <u>or</u> event B occurs.

Example

grad students $\cup$ women = all grads and female undergrads.

grad students $\cup$ freshmen = all students not sophomores, juniors, or seniors.

Complement

$\bar{A}$ = all outcomes where event A does <u>not</u> happen.

Example

$\overline{\text{grad students}}$ = undergrads

$\overline{(\text{grad students} \cup \text{women})} = \overline{\text{grad students}} \cap \overline{\text{women}}$ = male undergrads

Basic Axioms of Probability

ϕ = empty set (no outcome)

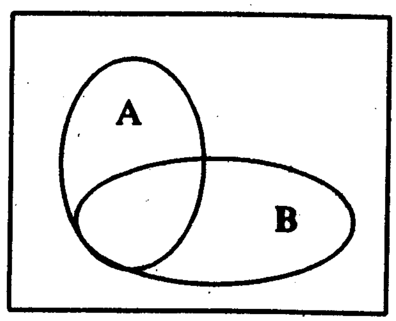
Ω = set of all outcomes

$P(\Omega)=1 \quad P(\phi)=0$

If $A \cap B = \phi$,

$P(A \cup B) = P(A) + P(B)$

Conditional probability



- $\mathrm{P}(A \mid B)$ = probability of A,
  given that B occurred
  - B is our new universe

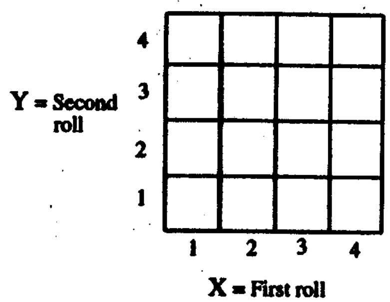
- **Definition:** Assuming $\mathrm{P}(B) \neq 0$,

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$$

Consequences: If $\mathrm{P}(A) \neq 0$, $P(B) \neq 0$, then

$$P(A \cap B) = P(B) \cdot P(A \mid B) = P(A) \cdot P(B \mid A)$$

Die roll example



- Let B be the event: $\min(X, Y) = 2$
- Let $M = \max(X, Y)$
- $P(M = 1 \mid B) =$
- $P(M = 2 \mid B) =$

Communication Channel

Binary Memoryless Channel (BMC) described by conditional probabilities

$$p(0 \text{ out} | 0 \text{ in}) = p_{00} = 1 - a$$

$$p(0 \text{ in}) = p_0$$

$$p(1 \text{ in}) = p_1$$

$$p(1 \text{ out} | 0 \text{ in}) = p_{01} = a$$

$$p(1 \text{ out} | 1 \text{ in}) = p_{11} = 1 - b$$

$$p(0 \text{ out} | 1 \text{ in}) = p_{10} = b$$

channel error = $(0 \text{ in} \cap 1 \text{ out}) \cup (1 \text{ in} \cap 0_{\text{out}})$

P (channel error) =

Can you interpret channel outputs so that

P (interpretation error) < P (channel error) ?

Maximum A Posteriori Decision Rule (MAP Rule)

If p (1 sent | 1 received) > p (0 sent | 1 received),
whenever a 1 is received, decide a 1 was sent.

If p (1 sent | 1 received < p (0 sent | 1 received)
whenever a 1 is received, decide a 0 was sent.

When do you flip the received bit?

Double Redundancy

Transmit 00 or 11, receive 00, 01, 10 or 11.

Have previously thought of this is as a single error detection scheme, no correction.

Now probability lets us interpret ambiguous outputs (01 and 10) in a statistically optimum way (MAP)

$$P(00 \text{ out } | 00 \text{ in}) = (1-a)^2$$

$$p(01 \text{ out } | 00 \text{ in}) = p(10 \text{ out } | 00 \text{ in}) = a(1-a)$$

$$p(11 \text{ out} | 00 \text{ in}) = a^2$$

$$p(00 \text{ in}) = p(0 \text{ in}) = p_0$$

$$p(11 \text{ in}) = p(1 \text{ in}) = p_1$$

$$p(11 \text{ out} | 11 \text{ in}) = (1-b)^2$$

$$p(01 \text{ out} | 11 \text{ in}) = p(10 \text{ out} | 11 \text{ in}) = b(1-b)$$

$$p(00 \text{ out} | 11 \text{ in}) = b^2$$

Problem

Suppose

$$p_0 = 0.7$$
$$p_1 = 0.3$$
$$a = 0.01$$
$$b = 0.1$$

Find the optimal (MAP) strategy for interpreting the outputs 00, 01, 10 and 11.