Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
Department of Mechanical Engineering

6.050J/2.110J **Information and Entropy** **Spring 2004**

Problem Set 10 Solutions

Solution to Problem 1: Well, Well, Well

Solution to Problem 1, part a.

Inside the well V(x) = 0 and therefore

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} \tag{10–6}$$

Solution to Problem 1, part b.

If E has a nonzero imaginary part E_{imag}, then the magnitude of $f(t)$ is a function of time, in particular

$$\mid f(t) \mid = exp(E_{imag} t/\hbar) \tag{10–7}$$

If $E_{imag} > 0$ then $\mid f(t) \mid$ gets large for large values of t (i.e., it blows up at infinity). If $E_{imag} < 0$ then $\mid f(t) \mid$ gets large for large values of $-t$ (i.e., it blows up at negative infinity). In either case it is impossible to normalize $\psi(x)$.

Solution to Problem 1, part c.

$$E\phi(x) = -\frac{\hbar^2}{2m} \frac{\partial^2 \phi(x,t)}{\partial x^2} \tag{10–8}$$

Solution to Problem 1, part d.

Since

$$\begin{aligned}
\phi(x) &= a\sin(kx) + b\cos(kx) && (10–9)\\
\frac{d\phi(x)}{dx} &= ak\cos(kx) - bk\sin(kx) && (10–10)\\
\frac{d^2\phi(x)}{dx^2} &= -ak^2\sin(kx) - bk^2\cos(kx) &&\\
&= -k^2\phi(x) && (10–11)
\end{aligned}$$

Therefore

$$E\phi(x) = \left(\frac{\hbar^2 k^2}{2m}\right)\phi(x) \tag{10–12}$$

so

$$E = \frac{\hbar^2 k^2}{2m} \tag{10–13}$$

Solution to Problem 1, part e.

One of the boundary conditions is $\phi(-L) = 0$, so

$$\begin{aligned} 0 &= \phi(-L) \\ &= asin(-kL) + bcos(-kL) \end{aligned} \tag{10–14}$$

The other boundary condition is $\phi(L) = 0$, so

$$\begin{aligned} 0 &= \phi(L) \\ &= asin(kL) + bcos(kL) \end{aligned} \tag{10–15}$$

From this we can determine that when a is nonzero, b must be zero, and vice-versa.

Solution to Problem 1, part f.

$\phi(x)$ must be zero at the boundaries, which implies

$$k = \frac{j\pi}{2L} \tag{10–16}$$

For odd j, the cosine term is zero, so a must be zero. For even j, b must be zero.

Solution to Problem 1, part g.

$$e_j = \frac{\hbar^2 \pi^2 j^2}{8mL^2} \tag{10–17}$$

Solution to Problem 1, part h.

$$\phi_j(x) = \begin{cases} b\cos\left(\frac{j\pi x}{2L}\right) & j \text{ odd} \\ a\sin\left(\frac{j\pi x}{2L}\right) & j \text{ even} \end{cases} \tag{10–18}$$

Solution to Problem 1, part i.

$$e_1 = \frac{\hbar^2 \pi^2}{8mL^2} \tag{10–19}$$

Solution to Problem 1, part j.

$$e_2 = \frac{\hbar^2 \pi^2}{2mL^2} \tag{10–20}$$

Solution to Problem 1, part k.

$$e_1 = \frac{\hbar^2 \pi^2}{2mL^2} \tag{10–21}$$

$$= \frac{(1.054 \times 10^{-34}\text{Joule-seconds})^2 \times (3.1416)^2}{8 \times (9.109 \times 10^{-31}\text{kilograms}) \times (2 \times 10^{-8}\text{meters})^2} \tag{10–22}$$

$$= 3.765 \times 10^{-23}\text{Joules} \tag{10–23}$$

(10–24)

Solution to Problem 1, part l.

Express this ground-state energy in electron-volts (1 eV= 1.602×10^{-19} Joules).

$$\begin{aligned} e_1 &= 3.765 \times 10^{-23} \text{Joules} \\ &= 2.347 \times 10^{-4} \text{eV} \end{aligned} \tag{10–25}$$