Preface

These notes, and the related freshman-level course, are about information. Although you may have a general idea of what information is and may have dealt with both precise and imprecise information, you may not have realized that the information you deal with can be quantified. Yes, that's right, you can measure the amount of information in a variety of situations and state general principles about how information behaves. We will consider applications to computation and to communications, and we will also look at general laws in other fields of science and engineering.

One of these general laws is the Second Law of Thermodynamics. Although thermodynamics, a branch of physics, deals with physical systems, the Second Law is approached here as an example of information processing in natural and engineered systems. The Second Law as traditionally stated in thermodynamics governs a physical quantity known as "entropy." Everybody has heard of entropy, but few really understand it. The Second Law is surely one of science's most glorious achievements, but as usually taught, through physical systems and models such as ideal gases, it is difficult to appreciate at an elementary level. On the other hand, the form of the Second Law that applies to computation and communications is more easily understood, especially today as the information revolution is getting under way.

These notes and the course based on them are intended for first-year university students. Although developed at the Massachusetts Institute of Technology, a university that specializes in science and engineering, and one at which a full year of calculus is required, not much use is made of calculus in these notes. Most examples are from discrete systems, not continuous systems, so that sums and differences can be used instead of integrals and derivatives. Thus this course should be accessible to students who are not studying engineering or science. In fact, when combined with a similar one about energy, this course could provide an excellent science background for liberal-arts majors.

The analogy between information and energy is interesting. In both high-school and first-year college physics courses, students learn that there is a physical quantity known as energy which is conserved (it cannot be created or destroyed), but which can appear in various forms (potential, kinetic, electric, chemical, etc.). Energy can exist in one region of space or another, can flow from here to there, can be stored for later use, and can be converted from one form to another. In many ways the industrial revolution was all about man's ability to harness energy for useful purposes. Energy is conserved – despite its being moved, stored, and converted, at the end of the day there was still exactly the same total amount of energy.

In the context of thermodynamics, this conservation of energy principle is known as the First Law. It has proven to be so important and fundamental that whenever a "leak" was found, the theory was rescued by defining a new form of energy. One example of this occurred in 1905 when Albert Einstein recognized that mass is a form of energy, as expressed by his famous formula $E = mc^2$. That understanding later enabled the development of devices (atomic bombs and nuclear power plants) that convert energy from its form as mass to other forms.

The corresponding unified theory of information and entropy is not as well developed, for several historical reasons. Yet such a theory is exactly what is needed to simplify the teaching of the concepts. The basic

notions can be appreciated, even at the freshman level, and applied to several domains.

These notes present such a unified view of information, in which entropy is one kind of information, and there are other kinds as well. Like energy, information can reside in one place or another, it can be transmitted through space, and it can be stored for later use. But unlike energy, information is inherently subjective, because it deals with what you know and what you don't know (entropy, as one form of information, is also subjective – this point makes some physicists uneasy). Also, information is not conserved as is energy; the Second Law states that entropy never decreases as time goes on – it generally increases and may only in limiting cases remain constant.

The unified framework presented here has never been developed specifically for freshmen. It is not entirely consistent with conventional thinking in various disciplines, which were developed separately. In fact, we may not yet have it right. One consequence of teaching at an elementary level is that unanswered questions close at hand are disturbing, and their resolution demands new research into the fundamentals. Trying to explain things rigorously but simply often requires new organizing principles and new approaches. In the present case, the new approach is to start with information and work from there to entropy, and the new organizing principle is the unified theory of information.

This will be an exciting journey. Welcome aboard!