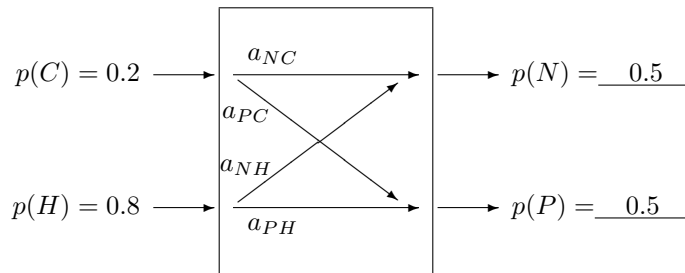


## Quiz Solutions

### Solution to **Problem 1: Hot Stuff (35%)**

Solution to Problem 1, part a.

Complete the following figure by indicating the transition probabilities and the output probabilities.



$a_{PC} \underline{0} \quad a_{NH} \underline{0.375} \quad a_{NC} \underline{1} \quad a_{PH} \underline{0.625}$

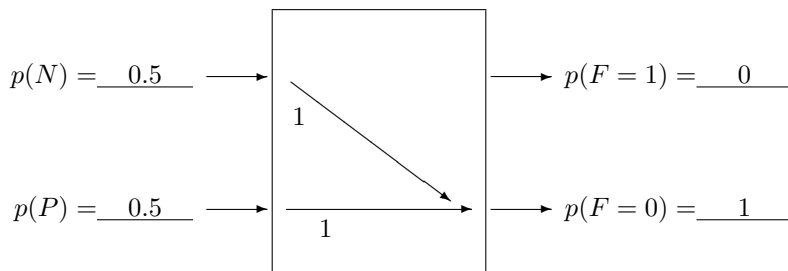
Solution to Problem 1, part b.

Your knowledge of information and entropy helps you give the input information  $I_{in}$ , output information  $I_{out}$ , noise  $N$ , loss  $L$ , and mutual information  $M$  of the sensor, all in bits.

$I_{in} = \underline{0.72} \quad L = \underline{0.485} \quad M = \underline{0.235} \quad N = \underline{0.765} \quad I_{out} = \underline{1}$

Solution to Problem 1, part c.

Add the transition arrows and probabilities to the diagram below, and insert the probabilities at the input and output (only put in transition arrows with nonzero probabilities).



**Solution to Problem 1, part d.**

What is the input information, loss, mutual information, noise, and output information of Ben's program in bits?

$$I_{in} = \underline{1} \quad L = \underline{1} \quad M = \underline{0} \quad N = \underline{0} \quad I_{out} = \underline{0}$$

**Solution to Problem 1, part e.**

Explain in words why the customers might have complained. The furnace never turned on.

---

**Solution to Problem 2: A Pretty Bad Channel (30%)****Solution to Problem 2, part a.**

Find the channel capacity of this channel in bits per second.

$$C = \underline{0.19 \text{ bits/second}}$$

**Solution to Problem 2, part b.**

Find the probability of zero, one, two, or three errors when a sequence of three bits is sent through the channel (assume the errors are independent).

$$p(0 \text{ errors}) \underline{27/64} \quad p(1 \text{ error}) \underline{27/64} \quad p(2 \text{ errors}) \underline{9/64} \quad p(3 \text{ errors}) \underline{1/64}$$

**Solution to Problem 2, part c.**

Give the output of this decoder for each possible 3-bit input.

$$\begin{array}{l} 000 \rightarrow \underline{0} \quad 001 \rightarrow \underline{0} \quad 010 \rightarrow \underline{0} \quad 011 \rightarrow \underline{1} \\ 100 \rightarrow \underline{0} \quad 101 \rightarrow \underline{1} \quad 110 \rightarrow \underline{1} \quad 111 \rightarrow \underline{1} \end{array}$$

**Solution to Problem 2, part d.**

Find the probability that the complete system makes an error if a 0 is transmitted.

$$p(\text{error}|0) = \underline{10/64}$$


---

**Solution to Problem 3: Building Constraints (35%)****Solution to Problem 3, part a.**

In the absence of other information, what set of probabilities  $C$ ,  $E$ , and  $R$  express your knowledge without any additional assumptions?

$$C \underline{1/3} \quad E \underline{1/3} \quad R \underline{1/3}$$

**Solution to Problem 3, part b.**

You remember reading in *The Tech* that the average cost of all the new buildings is \$120M. Write a constraint equation that relates  $C$ ,  $E$ , and  $R$  that incorporates this new information.

Constraint Equation:  $50C + 200E + 300R = 120$

**Solution to Problem 3, part c.**

What values of  $C$ ,  $E$ , and  $R$  are compatible with this information and also the fact that  $C$ ,  $E$ , and  $R$  must add up to 1?

$C_{min}$  0.53 (8/15)  $C_{max}$  0.72 (18/25)  $E_{min}$  0  $E_{max}$  0.46 (7/15)  $R_{min}$  0  $R_{max}$  0.28 (7/25)

**Solution to Problem 3, part d.**

Write this formula as a function of any one of the three probabilities  $C$ ,  $E$ , and  $R$ .

$$\begin{aligned}
 S &= C \log_2 \left( \frac{1}{C} \right) + E \log_2 \left( \frac{1}{E} \right) + R \log_2 \left( \frac{1}{R} \right) \\
 S(C) &= C \log_2 \left( \frac{1}{C} \right) + \frac{180 - 250C}{100} \log_2 \left( \frac{100}{180 - 250C} \right) + \frac{150C - 80}{100} \log_2 \left( \frac{100}{150C - 80} \right) \\
 S(E) &= \frac{180 - 100E}{250} \log_2 \left( \frac{250}{180 - 100E} \right) + E \log_2 \left( \frac{1}{E} \right) + \frac{70 - 150E}{250} \log_2 \left( \frac{250}{70 - 150E} \right) \\
 S(R) &= \frac{80 + 100R}{150} \log_2 \left( \frac{150}{80 + 100R} \right) + \frac{70 - 250R}{150} \log_2 \left( \frac{150}{70 - 250R} \right) + R \log_2 \left( \frac{1}{R} \right)
 \end{aligned}$$

Equation for the Entropy: \_\_\_\_\_