Name: ____

Massachusetts Institute of Technology

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6.050J/2.110J **Information and Entropy** **Spring 2004**

Issued: April 12, 2004, 1:00 PM **Quiz** **Due: April 12, 2004, 2:00 PM**

Note: Please write your name at the top of each page in the space provided. The last page may be removed and used as a reference table for the calculation of logarithms.

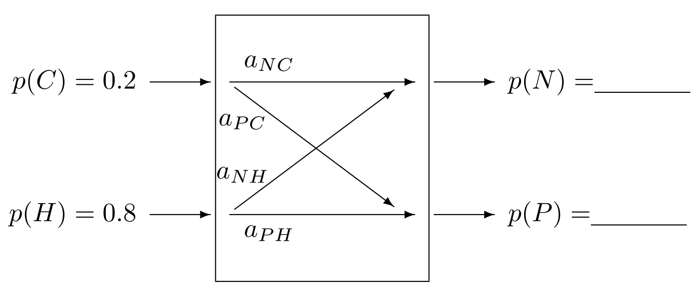
Problem 1: Hot Stuff (35%)

The NanoFurnace Company makes heaters for assembly lines. Their product looks at objects passing by, and warms them up if they are too cold. The furnace control system has two parts. First there is a sensor, which is supposed to output a negative voltage N when the object is cold, and a positive voltage P if the object is hot. Then there is an inference program that sets the furnace command F to 1 if the furnace is needed and 0 otherwise.

Unfortunately the sensors are cheap but unreliable. Every cold object is recognized and leads to a negative voltage, but only 5/8 (62.5%) of the hot objects are correctly identified. Since for all the customers the NanoFurnace company serves, 80% of the objects are already hot enough, and 20% are cold, this results in a large number of errors.

NanoFurnace's programmer, Ben Bitdiddle, wrote the inference program so it would compensate for this problem. It takes as input whether the voltage is positive or negative, and produces a logical value 1 if and only if the object temperature is more likely to be cold than hot for this voltage level. Of course, Ben knew the probabilities of objects being hot and cold ($p(H) = 0.8$, $p(C) = 0.2$) and the various sensor probabilities (for example, $p(P|H) = 0.625$).

a. Some customers have complained and the company has asked you for help. Your first step is to model the sensor as a nondeterministic process, as in the diagram below. Complete the following figure by indicating the transition probabilities and the output probabilities.

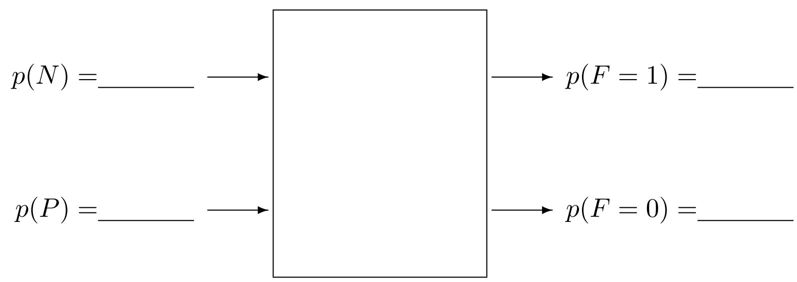


a_{PC} ________ a_{NH} ________ a_{NC} ________ a_{PH} ________

b. Your knowledge of information and entropy helps you give the input information I_{in}, output information I_{out}, noise N, loss L, and mutual information M of the sensor, all in bits.

I_{in} =________ L =________ M =________ N =________ I_{out} =________

c. You also figure out what Ben's program tells the furnace when a positive voltage is observed, and when a negative voltage is observed. Model this program as a process with possible inputs P and N, and output F. Add the transition arrows and probabilities to the diagram below, and insert the probabilities at the input and output (only put in transition arrows with nonzero probabilities).



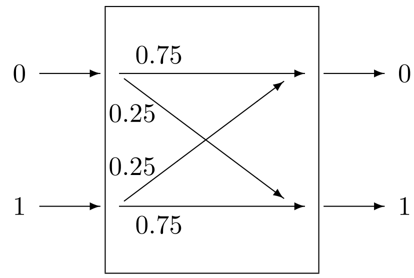
d. What is the input information, loss, mutual information, noise, and output information of Ben's program in bits?

I_{in} =________ L =________ M =________ N =________ I_{out} =________

e. Explain in words why the customers might have complained.

Problem 2: A Pretty Bad Channel (30%)

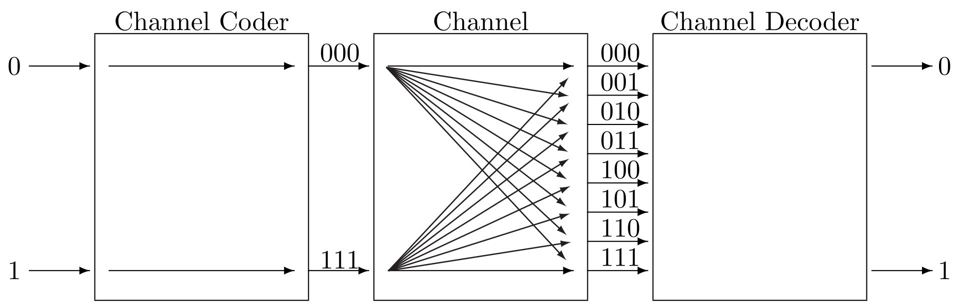
A communication channel can transmit 1 bit per second with the error probabilities given in the figure below. (This is the binary symmetric channel we have studied, with error probabilities that are independent from one transmission to another.)



a. Find the channel capacity of this channel in bits per second.

$C =$_______

To reduce the probability of error we introduce triple redundancy coding at the transmitter: a zero is encoded as '000' and a 1 is encoded as '111'. A corresponding decoder is installed at the receiver end.



b. Find the probability of zero, one, two, or three errors when a sequence of three bits is sent through the channel (assume the errors are independent).

p(0 errors)_______ p(1 error)_______ p(2 errors)_______ p(3 errors)_______

c. The decoder is a single-error-correcting decoder: it puts out the correct bit that was transmitted if there is either 0 or 1 error in transmission through the channel. Give the output of this decoder for each possible 3-bit input.

000→_______ 001→_______ 010→_______ 011→_______
100→_______ 101→_______ 110→_______ 111→_______

d. Find the probability that the complete system makes an error if a 0 is transmitted.

$p(\text{error}|0) =$ _______

Problem 3: Building Constraints (35%)

MIT has gone through a spurt of growth recently. There are so many new buildings that you have lost track. There are three types of buildings: cheap (cost \$50M), expensive (cost \$200M), and ridiculous (cost \$300M).

Your relative visit and you give them a tour of campus. While showing them around, you stop in front of one of these new buildings. They are curious how much it cost, but since you don't know in detail, so you use the principles you learned in Information and Entropy to help you cope with your uncertainty. You use probabilities to express your state of knowledge. Let C, E, and R stand for the probabilities the building is cheap, expensive, or ridiculous.

a. In the absence of other information, what set of probabilities C, E, and R express your knowledge without any additional assumptions?

C______ E______ R______

b. You remember reading in *The Tech* that the average cost of all the new buildings is \$120M. Write a constraint equation that relates C, E, and R that incorporates this new information.

Constraint Equation: ______________________________

c. What values of C, E, and R are compatible with this information and also the fact that C, E, and R must add up to 1?

C_{min}______ C_{max}______ E_{min}______ E_{max}______ R_{min}______ R_{max}______

d. You know that the probabilities that are consistent with your knowledge but have no additional assumptions is found by expressing your uncertainty as a function of one of the probabilities and then finding where its maximum occurs. Write this formula as a function of any one of the three probabilities C, E, and R. (You do not need to find this maximum point.)

Equation for the Entropy: ______________________________

Logarithm and Entropy Table

This page is provided so that you may rip it off the quiz to use as a separate reference table. In Table Q–1, the entropy $S = p\log(1/p) + (1-p)\log_2(1/(1-p))$.

p	1/8	1/5	1/4	3/10	1/3	3/8	2/5	1/2	3/5	5/8	2/3	7/10	3/4	4/5	7/8
$\log_2(1/p)$	3.00	2.32	2.00	1.74	1.58	1.42	1.32	1.00	0.74	0.68	0.58	0.51	0.42	0.32	0.18
S	0.54	0.72	0.81	0.88	0.92	0.95	0.97	1.00	0.97	0.95	0.92	0.88	0.81	0.72	0.54

Table Q–1: Table of logarithms in base 2 and entropy in bits