6.050J/2.110J **Information and Entropy** **Spring 2003**

Issued: April 25, 2003

Problem Set 10

Due: May 2, 2003

Problem 1: Entropy Goes Up

This problem is based on the magnetic dipole model in Chapter 12 of the notes. The system is assumed to have one dipole, as pictured, and for the purposes of this problem there is only one environment and it also contains exactly one dipole (this is to keep the calculations simple). For this problem you may assume that the environment and system have the same applied magnetic field.

The configuration is set up with the system having a low energy and the environment a high energy, and then the two are allowed to interact with the result that some energy in the form of heat flows from the environment to the system. You will calculate the amount of heat and the entropy before and after this operation.

The configuration is shown in Figure 10–1.

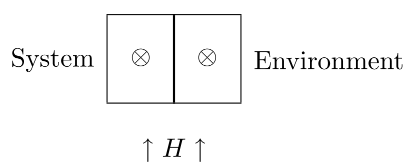

Figure 10–1: Dipole moment example.

Each of the dipoles shown can be either aligned with the field, in which case it contributes an energy $-m_d H$ or in the opposite direction, in which case it contributes $m_d H$. Thus the system has two states, one with energy $m_d H$, and one with energy $-m_d H$. The environment also has two states, with the same energies. As the problem starts, the two (system and environment) are isolated and each has a probability distribution as shown in these tables.

System				Environment			
State	Dipole	Energy $E_{s,i}(H)$	Probability $p_{s,i}$	State	Dipole	Energy $E_{e,j}(H)$	Probability $p_{e,j}$
i=0	up	$-m_d H$	0.45	$j = 0$	up	$-m_d H$	0.75
i=1	down	$m_d H$	0.55	$j = 1$	down	$m_d H$	0.25

Table 10–1: System and Environment Parameters

a. Find the system energy E_s (the expected value of the energy). You may leave your answer (and the other energies asked for) as a multiple of $m_d H$.

b. Find the environment energy E_e.

c. Find the environment entropy S_e. You may leave your answer (and the other entropies asked for) in terms of k_B.

d. Find the system entropy S_s.

Now consider what happens when the barrier between the system and the environment is removed, so they can interact. After some time has passed, your knowledge of the initial probabilities and separate energies is no longer relevant, and you can only treat the system as a whole. The Principle of Maximum Entropy can be used to estimate the probabilities of the four possible states (up-up, up-down, down-up, and down-down) without your assuming any information you do not have.

e. What is the total energy E_t?

f. Find β_t.

g. Find the four probabilities $p_{t,i,j}$. Hint: these are relatively simple. If you get numbers that seem random you have probably made a mistake.

h. Calculate the total entropy S_t and compare it to the sum of the system and environment entropies before the interaction.

i. Calculate the energy E_s in the system (the expected value). Hint: to do this, you will need to infer from the four probabilities for the total configuration $p_{t,i,j}$ the probabilities for the two system states $p_{s,i}$.

j. How much energy came into the system from the environment in the form of heat during the interaction?

Problem 2: Energy Conversion (the charge pump)

A startup company has developed a product that can charge a battery using mechanically supplied energy. You are asked to analyze the device and determine whether it will work, and if so how much energy can be converted. (Note, this problem does not involve either information or entropy, but it does use an approach to energy conversion that will be used in describing heat engines and the second law of thermodynamics.)

The device is shown below. It consists of a stationary conducting plate of area A square meters with negative charge (the minus signs) $-q$ coulombs and a movable plate the same size a distance d meters away, with positive charge $+q$ coulombs (the plus signs). The movable plate travels vertically along frictionless guides (not shown), and stays parallel to the stationary plate. It can get as close as d_{min} meters to the stationary plate and as far as d_{max} meters away. The top plate is moved by a mechanical force which balances the attractive force of the two electrodes (remember that unlike charges attract). The top electrode is wired to a three-way switch that can connect it to a low-voltage battery with voltage V_{low} volts, a high-voltage battery with voltage V_{high} volts, or neither.

Your knowledge of electrostatics enables you to deduce the following.

- The charge q is related to voltage v between the plates by the formula $q = \epsilon_0 Av/d$ where ϵ_0 is the permittivity of free space, 8.854×10^{-12} farads per meter.
- The attractive force between the plates (balanced by the applied force upward on the top plate) is $f = qv/2d$.
- The charge q can only change if the switch connects the top plate with one of the batteries, in which case the energy supplied by the battery to change the charge from q_1 to q_2 is $v(q_2 - q_1)$.

The user's manual for this product describes the operation of charging the high-voltage battery as follows.

1. Start with the low-voltage battery connected and the plates at their minimum separation, d_{min}.
2. Throw the switch to the middle position so the charge cannot change.

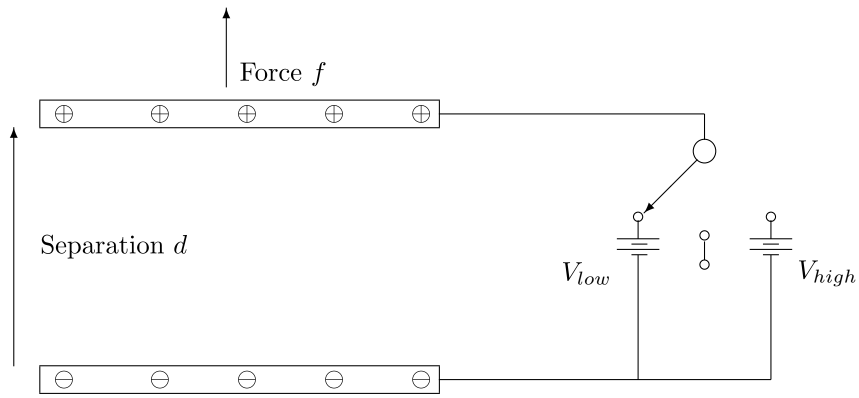

Figure 10–2: Charge pump

3. Pull the plates apart, carefully noting the voltage on the built-in voltmeter (not shown). The voltage will rise. Continue until the voltage is equal to V_{high}.
4. Throw the switch so that the high-voltage battery is connected. Since the voltage is equal to the voltage of that battery this operation does not by itself affect q.
5. Continue to pull the plates apart to their maximum separation d_{max}. This causes the charge q to decrease, meaning charge is being delivered to the high-voltage battery.
6. Throw the switch to the middle position.
7. Reduce the applied force to allow the top plate to move downward until the voltage reaches V_{low}.
8. Throw the switch to connect to the low-voltage battery.
9. Continue to let the plates come closer together until $d = d_{min}$. The location of the plate and the charge are what they were at the beginning of these instructions, and some charge has been placed on the high-voltage battery thereby charging it up, and the same amount of charge has been taken from the low-voltage battery, thereby discharging it.
10. Repeat steps 2 - 9 as required to obtain the desired battery charge.

Naturally you want to know how much energy has been added to the high-voltage battery and how much was lost from the low-voltage battery, and therefore how much was supplied by whatever applied the mechanical force. You decide to ignore the effects of gravity.

a. Draw this charging cycle as a rectangle in the charge-voltage plane (charge on the vertical axis, voltage on the horizontal).

b. Mark the parts of this diagram that correspond to the numbered instructions in the user's manual.

c. Indicate for each of the four legs whether mechanical energy is being supplied to the device or taken from the device, and whether electrical energy is being supplied to or taken from either of the batteries.

d. Find a formula for the charge q_0 delivered to the high-voltage battery in one cycle as a function of d_{min}, $_{dmax}$, V_{low}, V_{high}, ϵ_0, and A.

e. Find the energy supplied to the high-voltage battery per cycle (this is the product of its voltage times the charge supplied).

f. Find the energy delivered by the low-voltage battery per cycle.

g. Find the energy supplied by the mechanical source per cycle.

h. By taking the device apart you discover that the plates are square, 4 cm on a side, and the movable plate can travel to within 0.1 mm of the fixed plate, or as far away as 5.0 mm. You guess that a person could operate the device at a rate of two charging cycles per second. You want to charge a 9-volt storage battery using a 1.5-volt battery for the low voltage. How long would it take to charge it with 10^{-9} coulombs (enough to run a 1 nanowatt load for 9 seconds)?

Turning in Your Solutions

You may turn in this problem set by e-mailing your written solutions, M-files, and diary to 6.050-submit@mit.edu. Do this either by attaching them to the e-mail as *text* files, or by pasting their content directly into the body of the e-mail (if you do the latter, please indicate clearly where each file begins and ends). If you have figures or diagrams you may include them as graphics files (GIF, JPG or PDF preferred) attached to your email. Alternatively, you may turn in your solutions on paper in room 38-344. The deadline for submission is the same no matter which option you choose.

Your solutions are due 5:00 PM on Friday, May 2, 2003. Later that day, solutions will be posted on the course website.