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6.050J/2.110J **Information and Entropy** **Spring 2003**

Issued: April 24, 2003, 12:00 PM **Quiz** **Due: April 24, 2003, 1:00 PM**

Note: Please write your name at the top of each page in the space provided. The last page may be removed and used as a reference table for the calculation of logarithms.

Problem 1: Digital Devices (20%)

You have a large number of *AND* and *OR* gates and want to use them as components in a new type of two-input, two-output gate. The two inputs A and B of the NEW gate are fed directly to the inputs of the *AND* and *OR* gates. The outputs X and Y of the NEW gate are the outputs of the *AND* and *OR* gates, respectively.

a. Draw the transition diagram of this gate by filling in the diagram below.

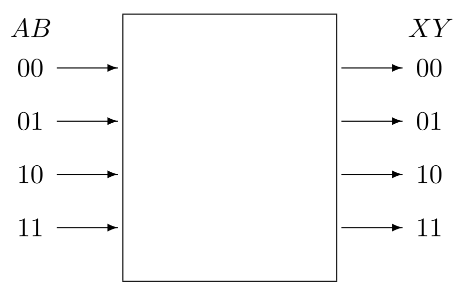

Figure Q–1: The NEW gate

b. Your boss hears that this gate is lossy, and is worried about the consequences for the company's public image from producing a gate with such a negative attribute. He asks you to define a set of constraints on the probabilities for the inputs AB for which the new gate is lossless. Don't make the constraints more severe than necessary.

Constraints____

Problem 2: The Price is Right (40%)

Fly-By-Night Airlines has started an overnight service between Boston and San Francisco. It sells tickets to individuals for \$100.00, to groups for \$50.00 each, and to corporate travellers for \$300.00. Nobody can tell by looking at the passengers how much they paid for their tickets.

On one of their flights, you wonder about the person sitting next to you. You wonder whether she paid at an group, individual, or corporate rate. Naturally, you express your uncertainty by assigning probabilities (call them G, I, and C). Without any information, all you can do is assign the probabilities of each to be one-third.

a. With these probabilities, what is your uncertainty (or entropy) expressed in bits?

$S =$ ______________________________

You read in the airline magazine a statement from the president of the company that the average selling price of a ticket is \$150.00. Naturally you want to modify your probabilities G, I, and C to make them consistent with this new knowledge.

b. What values of G, I, and C are possible?

I_{min} ________ I_{max} ________ G_{min} ________ G_{max} ________ C_{min} ________ C_{max} ________

In order to estimate the probabilities, you decide to use the Principle of Maximum Entropy. Express the entropy as a function of a single probability (any one of the three will do). You do not need to actually solve for the probabilities that maximize the entropy.

c. What is the equation for the entropy?

$S =$ ______________________________

You overhear the staff saying that this flight, as usual, consists of 30% corporate passengers. With this added information, you are able to further refine your estimates. You know that $C_{min} = C_{max} = 0.30$.

d. What values of G and I are now possible?

G_{min} __________ G_{max} __________ I_{min} __________ I_{max} __________

e. What is the entropy?

$S =$ ______________________________

Problem 3: Corporate Pricing Procedure (40%)

At an earlier time, Fly-By-Night had a smaller corporate customer base. It had 25% group passengers, 50% individual, and 25% corporate passengers. One day a clerk, Betty Bilker, in the billing department recommended a change in procedure which increased the average amount paid per ticket, and earned for her the coveted "employee of the month" award. She simply added a few lines of code to the billing software so that half of the individual tickets, selected at random, were billed at the corporate rate. All corporate and group tickets were billed correctly. The correct rates were $R_G = \$50$, $R_I = \$100$, and $R_C = \$300$.

a. After Betty changes the billing software, what is the probability that any particular rate is charged?

$p(R_G) =$ ______________ $p(R_I) =$ ______________ $p(R_C) =$ ______________

b. What was the average ticket bill before and after this innovative change?

Before ______________________ After ______________________

Upon hearing about this episode, you decide to model this program as a non-determinisitic process. You consider the input to this process to be the events of a customer purchasing one of the three classes of tickets (G, I, or C), and the output as the events corresponding to the amounts of the bill (R_G, R_I, or R_C).

c. Put the transition probabilities in the diagram below.

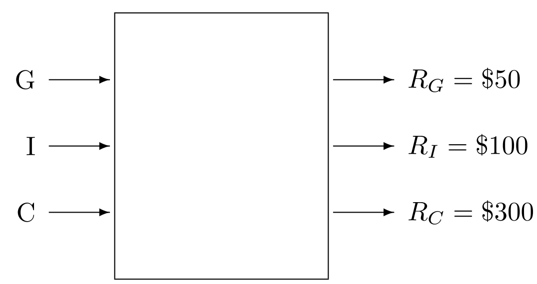

Figure Q–2: Fly-By-Night Transition Diagram

You wonder whether you can infer anything about the ticket category from knowing the billed amount, so you make a chart showing, for each billed amount, the probability of each of the ticket categories. For example, if the bill is for \$300, the probability that the ticket is a group ticket is $p(G \mid 300)$, which is zero.

d. Fill in Table Q–1. We have already given you one of the entries.

$R =$	R_G (\$50)	R_I(\$100)	R_C(\$300)
$p(G \mid R)$			0
$p(I \mid R)$			
$p(C \mid R)$			

Table Q–1: Fly-By-Night Price to Ticket Transition Table

e. Viewing this process as a communication channel, what is the input information, the output information, the noise, loss, and mutual information?

$I_{in} =$ ___________ $I_{out} =$ ___________ $N =$ ___________ $L =$ ___________ $M =$ ___________

Logarithm and Entropy Table

This page is provided so that you may rip it off the quiz to use as a separate reference table. In Table Q–2, the entropy $S = p\log(1/p) + (1-p)\log_2(1/(1-p))$.

p	1/8	1/5	1/4	3/10	1/3	3/8	2/5	1/2	3/5	5/8	2/3	7/10	3/4	4/5	7/8
$\log_2(1/p)$	3.00	2.32	2.00	1.74	1.58	1.42	1.32	1.00	0.74	0.68	0.58	0.51	0.42	0.32	0.18
S	0.54	0.72	0.81	0.88	0.92	0.95	0.97	1.00	0.97	0.95	0.92	0.88	0.81	0.72	0.54

Table Q–2: Table of logarithms in base 2 and entropy in bits